Lecture Notes, Part II
Economics 101, Section 104
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1 Welfare Analysis

In the first part of the class, I introduced some basic economic policies in which the government may engage in a market. We saw in the case of taxes, price ceilings, and some price floors, the quantity traded in the market falls; with a subsidy, the quantity traded rises. Price ceilings and subsidies cause the price consumers pay to fall. Taxes and price floors cause the price consumers pay to rise. The price sellers receive rises when there is a subsidy or price floor but falls when there is a price ceiling or tax. This all speaks to who benefits or loses with these policies. In this section, I will analyze more carefully the welfare consequences of these policies.

1.1 The Basics

Here I outline the basic tools of analysis, starting with some definitions. Let's start with the consumer side of things.

Definition. **Marginal Benefit:** The additional benefit a consumer gets from consuming another unit of the good.

- For unit i of a good, denote the marginal benefit as MB_i.
- This is measured in dollars per unit.
- It measures the additional amount the consumer is willing to pay to consume another unit of the good.

Suppose a consumer consumes Q units of the good. The **total benefit**, denoted TB, a consumer gets from consuming the Q units is the sum of the marginal benefits: $TB = \sum_{i=1}^{Q} MB_i$. Now, let the price demanders pay for unit i be P_{Di}. The consumer's total expenditures on the Q units are $TE = \sum_{i=1}^{Q} P_{Di}$. Note, if the price buyers pay is constant, i.e., $P_{Di} = P_D$ for all units, then consumer expenditures are $TE = P_D \cdot Q$

Definition. **Consumer Surplus:** The difference between the maximum amount a consumer is willing to pay for a good and the amount the consumer actually pays.

- For unit i, the consumer surplus is $CS_i = (MB_i - P_{Di})$.

- The consumer's total surplus on the Q units is $CS = \sum_{i=1}^{Q} CS_i = \sum_{i=1}^{Q} (MB_i - P_{Di}) = \sum_{i=1}^{Q} MB_i - \sum_{i=1}^{Q} P_{Di} = TB - TE.$

Suppose you go into Best Buy to purchase the latest electronic noise from Britney Spears. You've got \$50 to spend. You're marginal benefit from the first CD is \$20, and the price of the CD is \$15. Should you buy it? Yes, your consumer surplus from the first CD is \$5.

Suppose the marginal benefit from the second CD is \$17 (one for home, one for the car). The price is still \$15. Should you buy it? Yes, your consumer surplus from the second CD is \$2. Your total consumer surplus then is \$5+\$2=\$7.

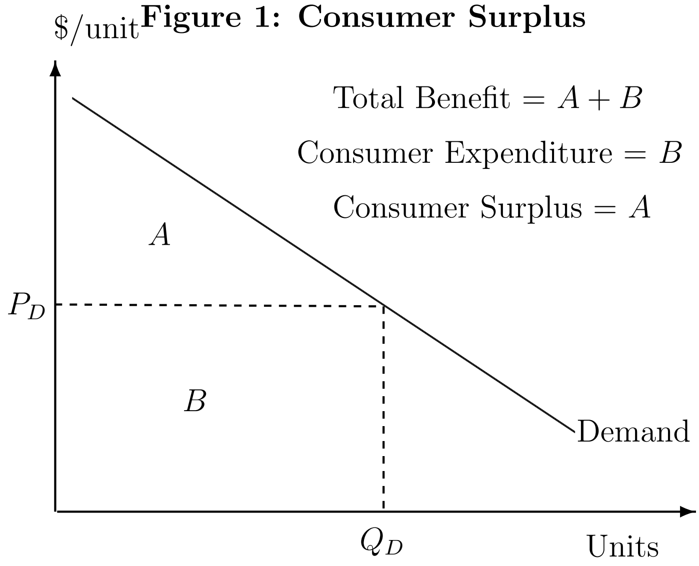
Suppose the marginal benefit from the third CD is \$10. The price is still \$15. Should you buy it? No, your consumer surplus from the third CD is −\$5. Your total consumer surplus if you bought the third CD would be \$5+\$2-\$5=\$2. You'd be worse off if you bought the third CD. Don't buy it!

This suggests that consumers should buy units of the good so long as the marginal benefit of the unit is at least as great as the extra expenditures required to purchase that unit, i.e., $MB_i \geq ME_i \Leftrightarrow MB_i \geq P_{Di}$. Doing so will maximize the consumer's surplus!

Marginal benefit is intimately tied to the consumer's demand curve. In fact, the second interpretation of the demand curve is that, for a given quantity, the demand curve tells us the maximum price the consumer is willing to pay for that unit. In this sense, it measures the consumer's marginal benefit of consuming that unit of the good.

- The area below the demand curve measures the consumer's total benefit of purchasing the good.
- The area below the price the consumer pays measures the consumer's expenditures on the good.
- Consumer surplus is the area below the demand curve and above the price consumers pay.

Figure 1: Consumer Surplus



Now I turn to the supply side of the market.

Definition. **Marginal Revenue (MR):** The extra revenue a firm gets from selling another unit of the good: $MR = \frac{\Delta TR}{\Delta Q}$.

- For unit i of a good, denote the marginal revenue as MR_i.
- This is measured in dollars per unit.

Suppose a firm produces Q units of the good. The total revenue a producer receives from producing the Q units is the sum of the marginal revenues: $TR = \sum_{i=1}^{Q} MR_i$.

Now, if the price producers receive is the same for each unit i it sells, say P_S, the producer's total revenue on the Q units is $TR = \sum_{i=1}^{Q} MR_i = \sum_{i=1}^{Q} P_S = P_S \cdot Q$.

Definition. **Fixed Cost (FC):** Those costs that do not vary with output.

Definition. **Variable Cost (VC):** Those costs that vary with output: $VC = VC(Q)$.

Definition. **Total Cost (TC):** The sum of fixed and variable costs: $TC(Q) = FC + VC(Q)$.

Definition. **Marginal Cost:** The extra cost associated with producing an extra unit of the good: $MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta(FC+VC)}{\Delta Q} = \frac{\Delta VC}{\Delta Q} + \frac{\Delta FC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}$ since $\frac{\Delta FC}{\Delta Q} = 0$ by definition.

- For unit i of a good, denote the marginal cost as MC_i.
- This is measured in dollars per unit of output.

The variable cost a producer incurs from producing the Q units is the sum of the marginal costs: $VC = \sum_{i=1}^{Q} MC_i$.

Definition. **Producer Surplus:** The benefit to firms from production in excess of the variable costs of production.

- For unit i, the producer surplus is $PS_i = (MR_i - MC_i)$.
- The producer's total surplus on the Q units is $PS = \sum_{i=1}^{Q} PS_i = \sum_{i=1}^{Q}(MR_i - MC_i) = \sum_{i=1}^{Q} MR_i - \sum_{i=1}^{Q} MC_i = TR - VC$.

Suppose you're producing Britney Spears's latest electronic noise. Suppose the market price for the CD is \$15. You're marginal cost of producing the first CD is \$10. Should you produce it? Yes, your producer surplus from the first CD is \$5.

Suppose the marginal cost of selling the second CD is \$13. Should you produce it? Yes, your producer surplus from the second CD is \$2. Your total producer surplus then is \$5+\$2=\$7.

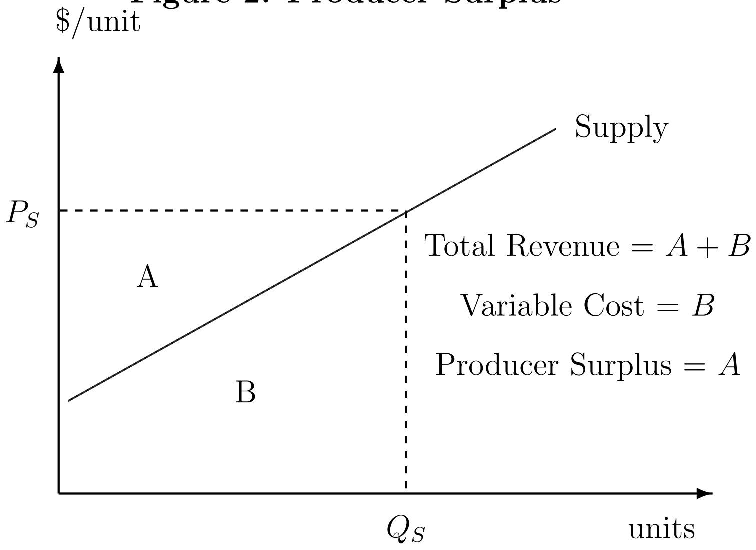
Suppose the marginal cost from selling the third CD is \$20. Should you produce it? No, your producer surplus from the third CD is $-\$5$. Your total producer surplus if you produced the third CD would be \$5+\$2-\$5=\$2. You'd be worse off if you produced the third CD. Don't produce it!

This suggests that producers should sell units of the good so long as the marginal revenue of the unit is at least as great as the marginal cost of producing that unit, i.e., $MR_i \geq MC_i$. With a constant price producers receive, this becomes $P_S \geq MC_i$. Doing so will maximize the producer's surplus!

Marginal cost is intimately tied to the firm's supply curve. Indeed, they are one in the same; the firm's marginal cost curve is the firm's supply curve (more on this later). The second interpretation of the supply curve is that, for a given quantity, the supply curve tells us the minimum price the firm is willing to receive for that unit. In this sense, it measures the producer's marginal cost of producing that unit of the good.

- The area below the price measures the firm's total revenue from selling the good.
- The area below the supply curve is the firm's variable costs of production.
- Producer surplus is the area below the price producers receive and above the supply curve.

Figure 2: Producer Surplus



Definition. **Gains from Trade:** Sum of producer and consumer surpluses.

Definition. **Welfare/Total Surplus:** For this class, our measure of welfare will be the total gains from trade plus tax revenues minus government payments.

Definition. **Deadweight Loss:** net loss of total surplus resulting from a market distortion.

As a preview of the results, taxes, binding price floors/ceilings, and quotas cause deadweight loss because too little of the good is produced. Subsidies cause deadweight loss because too much of the good is produced.

1.2 Price Controls

I will only consider the case of binding price floors and price ceilings because those are the interesting cases.

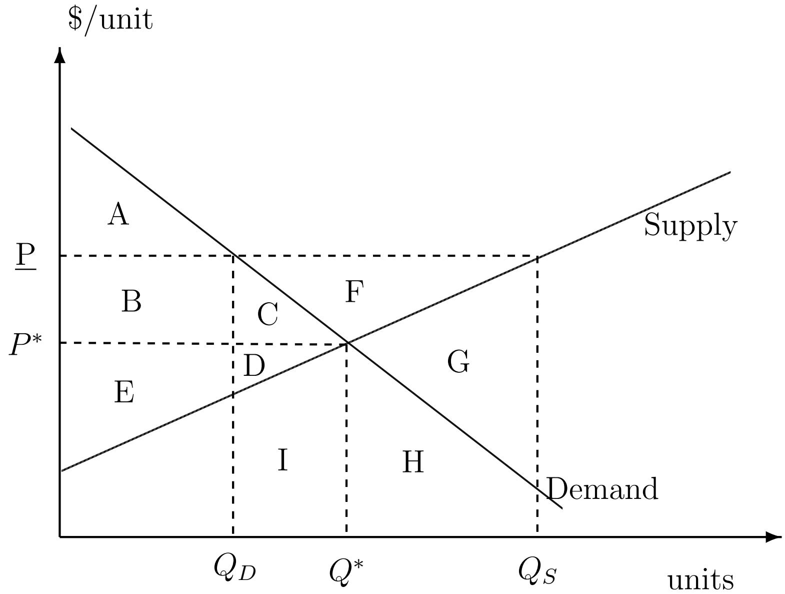
1.2.1 Price Floor

There are two cases to consider here, one in which the government agrees to buy any surplus units produced and one in which it doesn't. I examine each case in turn.

Case 1: Government agrees to buy surplus units

Consider Figure 3. With the price floor of $\underline{P}$, quantity supplied is Q_S, and quantity demanded by private consumers is Q_D. The government purchases the difference: $(Q_S - Q_D)$.

Figure 3: Price Floor, Government Buys Surplus



The following table outlines the changes in variables of interest. Producers are unambiguously better off because they receive a higher price per unit and sell more units. Consumers are unambiguously worse off because they pay a higher price per unit and purchase fewer units. Government expenditures are positive here. In total, society is worse off by $(C + D + G + H + I)$; the deadweight loss is $(C + D + G + H + I)$.

Variable	Before	After	Change
Price ($/unit)	P^*	$\underline{P}$	$(\underline{P} - P^*) > 0$
Quantity Traded (units)	Q^*	Q_S	$(Q_S - Q^*) > 0$
Consumer Expenditures ($)	$(P^* \cdot Q^*)$	$(\underline{P} \cdot Q_D)$	$[(\underline{P} \cdot Q_D) - (P^* \cdot Q^*)] \gtreqless 0$
Government Expenditures ($)	0	$\underline{P} \cdot (Q_S - Q_D)$	$\underline{P} \cdot (Q_S - Q_D) > 0$
Producer Revenue ($)	$(P^* \cdot Q^*)$	$(\underline{P} \cdot Q_S)$	$[(\underline{P} \cdot Q_S) - (P^* \cdot Q^*)] > 0$
Consumer Surplus ($)	$(A + B + C)$	A	$-(B + C) < 0$
Producer Surplus ($)	$(D + E)$	$(B + C + D + E + F)$	$(B + C + F) > 0$
Gains from Trade ($)	$(A + B + C + D + E)$	$(A + B + C + D + E + F)$	$F > 0$
Govt. Expenditures ($)	0	$(C + D + F + G + H + I)$	$(C + D + F + G + H + I) > 0$
Total Surplus ($)	$(A + B + C + D + E)$	$(A + B + E) - (G + H + I)$	$-(C + D + G + H + I) < 0$

Case 2: Government doesn't buy any surplus units

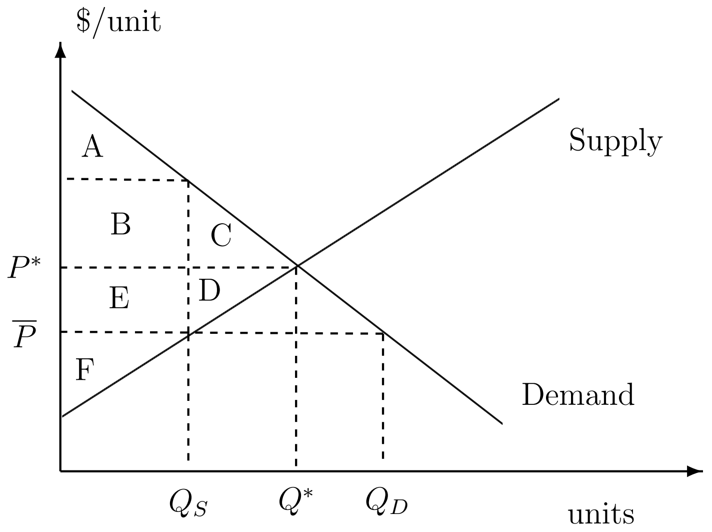
You can refer again to Figure 3. With the price floor of $\underline{P}$, firms would normally want to supply Q_S, but consumers demand only Q_D. Thus, Q_D units get produced and traded in the market. The following table outlines the changes in variables of interest. Consumers are unambiguously worse off because they pay a higher price per unit and purchase fewer units; consumer surplus falls by $(B + C)$. Producers may or may not be better off; the change in producer surplus is $(B - D)$. Those producers who end up selling the units are better off because they receive a higher price; but fewer units are traded, so the consumers who are no longer selling units are worse off. In total, society is worse off by $(C + D)$; the deadweight loss is $(C + D)$.

Variable	Before	After	Change
Price (\$/unit)	P^*	$\underline{P}$	$(\underline{P} - P^*) > 0$
Quantity Traded (units)	Q^*	Q_D	$(Q_D - Q^*) < 0$
Consumer Expenditures (\$)	$(P^* \cdot Q^*)$	$(\underline{P} \cdot Q_D)$	$[(\underline{P} \cdot Q_D) - (P^* \cdot Q^*)] \gtreqless 0$
Producer Revenue (\$)	$(P^* \cdot Q^*)$	$(\underline{P} \cdot Q_D)$	$[(\underline{P} \cdot Q_D) - (P^* \cdot Q^*)] \gtreqless 0$
Consumer Surplus (\$)	$(A + B + C)$	A	$-(B + C) < 0$
Producer Surplus (\$)	$(D + E)$	$(B + E)$	$(B - D) \gtreqless 0$
Gains from Trade (\$)	$(A + B + C$ $+D + E)$	$(A + B + E)$	$-(C + D) < 0$
Total Surplus (\$)	$(A + B + C$ $+D + E)$	$(A + B + E)$	$-(C + D) < 0$

1.2.2 Price Ceiling

With a price ceiling of $\overline{P}$, quantity demanded is Q_D, but quantity supplied is Q_S. Thus, only Q_S units are traded.

Figure 4: Price Ceiling



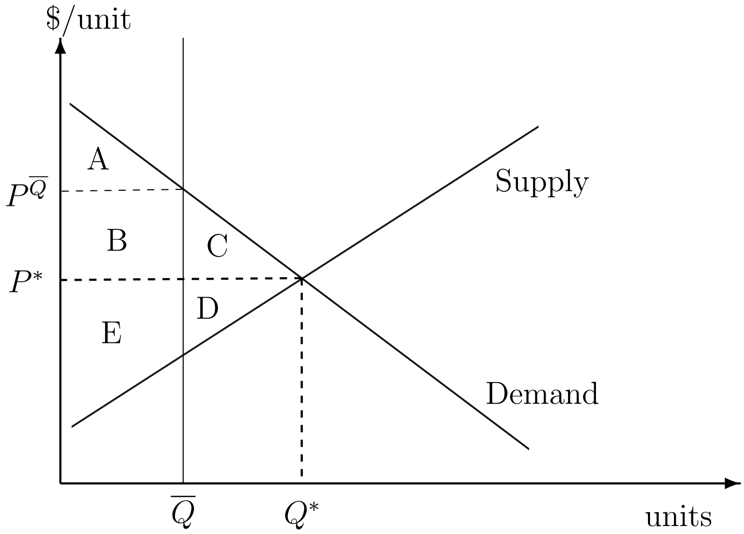
The following table outlines the changes in variables of interest. Producers are unambiguously worse off because they receive a lower price per unit and sell fewer units; producer surplus falls by $(D + E)$. Consumers may or may not be better off; the change in consumer surplus is $(E - C)$. Those consumers who end up purchasing the units are better off because they pay a lower price; but fewer units are traded, so the consumers who were purchasing the good before but aren't able to now are worse off. In total, society is worse off by $(C + D)$; the deadweight loss is $(C + D)$.

Variable	Before	After	Change
Price ($/unit)	P^*	$\overline{P}$	$(\overline{P} - P^*) < 0$
Quantity Traded (units)	Q^*	Q_S	$(Q_S - Q^*) < 0$
Consumer Expenditures ($)	$(P^* \cdot Q^*)$	$(\overline{P} \cdot Q_S)$	$[(\overline{P} \cdot Q_S) - (P^* \cdot Q^*)] < 0$
Producer Revenue ($)	$(P^* \cdot Q^*)$	$(\overline{P} \cdot Q_S)$	$[(\overline{P} \cdot Q_S) - (P^* \cdot Q^*)] < 0$
Consumer Surplus	$(A + B + C)$	$(A + B + E)$	$(E - C) \gtreqless 0$
Producer Surplus	$(D + E + F)$	F	$-(D + E) < 0$
Gains from Trade	$(A + B + C$ $+ D + E + F)$	$(A + B + E + F)$	$-(C + D) < 0$
Total Surplus	$(A + B + C$ $+ D + E + F)$	$(A + B + E + F)$	$-(C + D) < 0$

1.3 Quantity Controls

I will only consider the case of a binding quota. With a binding quota of $\overline{Q}$, in equilibrium, $\overline{Q}$ units are traded in the market at a price of $P^{\overline{Q}}$.

Figure 5: Quota



The following table outlines the changes in variables of interest. Consumers are unambiguously worse off because they pay a higher price per unit and purchase fewer units; consumer surplus falls by $(B + C)$. Producers may or may not be better off; the change in producer surplus is $(B - D)$. The producers that end up selling units are certainly better off because

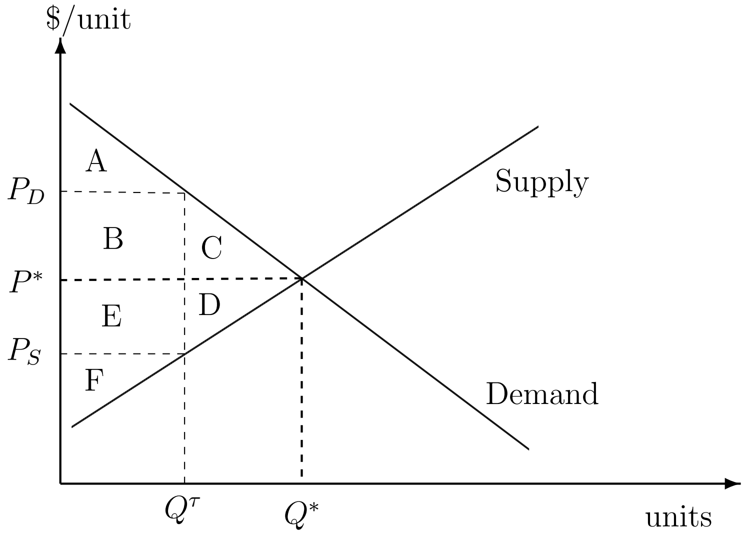
they receive a higher price per unit. However, since fewer units are sold, the firms that were selling before the quota but are not selling now are worse off. In total, society is worse off by $(C + D)$; the deadweight loss is $(C + D)$.

Variable	Before	After	Change
Price ($/unit)	P^*	$P^{\overline{Q}}$	$(P^{\overline{Q}} - P^*) > 0$
Quantity Traded (units)	Q^*	$\overline{Q}$	$(\overline{Q} - Q^*) < 0$
Consumer Expenditures ($)	$(P^* \cdot Q^*)$	$(P^{\overline{Q}} \cdot \overline{Q})$	$[(P^{\overline{Q}} \cdot \overline{Q}) - (P^* \cdot Q^*)] \gtreqless 0$
Producer Revenue ($)	$(P^* \cdot Q^*)$	$(P^{\overline{Q}} \cdot \overline{Q})$	$[(P^{\overline{Q}} \cdot \overline{Q}) - (P^* \cdot Q^*)] \gtreqless 0$
Consumer Surplus ($)	$(A + B + C)$	A	$-(B + C) < 0$
Producer Surplus ($)	$(D + E)$	$(B + E)$	$(B - D) \gtreqless 0$
Gains from Trade ($)	$(A + B + C$ $+D + E)$	$(A + B + E)$	$-(C + D) < 0$
Total Surplus ($)	$(A + B + C$ $+D + E)$	$(A + B + E)$	$-(C + D) < 0$

1.4 Taxes

I will only consider a per-unit tax of size $\$\tau$/unit. From before, we reasoned that the price demanders pay, P_D, is equal to the price sellers receive, P_S, plus the per unit tax, i.e., $P_D = P_S + \tau \Leftrightarrow \tau = P_D - P_S$. For simplicity, I assume that firms remit tax revenues to the government.

Figure 6: Per-unit Tax



The following table outlines the changes in variables of interest for a market that has an upward-sloping supply curve and a downward-sloping demand curve.

Variable	Before	After	Change
Demanders' Price (\$/unit)	P^*	P_D	$(P_D - P^*) > 0$
Sellers' Price (\$/unit)	P^*	P_S	$(P_S - P^*) < 0$
Quantity Traded (units)	Q^*	Q^τ	$(Q^\tau - Q^*) < 0$
Consumer Expenditures (\$)	$(P^* \cdot Q^*)$	$(P_D \cdot Q^\tau)$	$[(P_D \cdot Q^\tau) - (P^* \cdot Q^*)] \gtreqless 0$
Producer Revenue (\$)	$(P^* \cdot Q^*)$	$(P_S \cdot Q^\tau)$	$[(P_S \cdot Q^\tau) - (P^* \cdot Q^*)] < 0$
Government Revenue (\$)	0	$\tau \cdot Q^\tau$	$\tau \cdot Q^\tau > 0$
Consumer Surplus	$(A + B + C)$	A	$-(B + C) < 0$
Producer Surplus	$(D + E + F)$	F	$-(D + E) < 0$
Gains from Trade	$(A + B + C$ $+D + E + F)$	$(A + F)$	$-(B + C + D + E) < 0$
Government Revenue	0	$(B + E)$	$(B + E) > 0$
Total Surplus	$(A + B + C$ $+D + E + F)$	$(A + B$ $+E + F)$	$-(C + D) < 0$

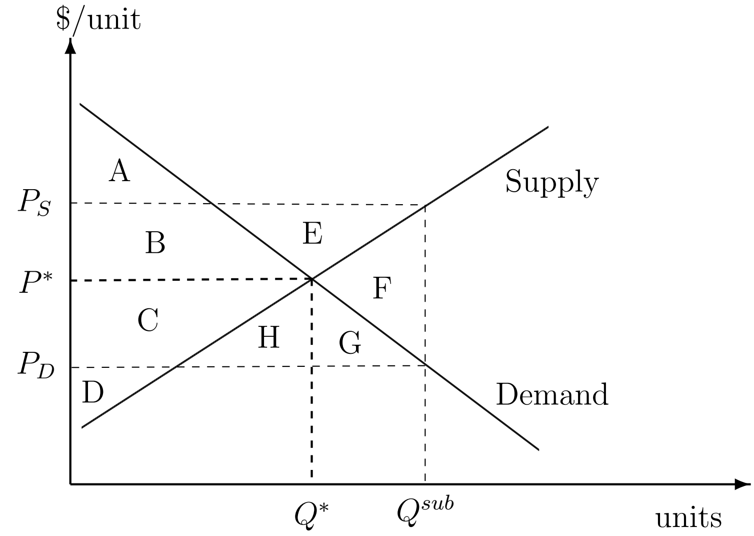
As a note, for a given supply curve, there will be a smaller deadweight loss the less elastic is the demand curve. Likewise, for a given demand curve, there will be a smaller deadweight loss the less elastic is the supply curve. This is because, for a given tax, holding the supply (demand) curve constant, the number of units traded under the tax will be greater the less elastic is the demand (supply) curve.

- For a given supply (demand) curve, there is no deadweight loss if demand (supply) is perfectly inelastic. Consumer surplus (producer surplus) goes down by exactly the amount of the tax revenue raised.

1.5 Subsidies

I will only consider a per-unit subsidy of size \$$s$/unit. From before, we reasoned that the price sellers receive, P_S, is equal to the price demanders pay, P_D, plus the per unit subsidy, i.e., $P_S = P_D + s \Leftrightarrow s = P_S - P_D$. For simplicity, I assume that firms receive the subsidy payments from the government.

Figure 7: Per-unit Subsidy



The following table outlines the changes in variables of interest. Consumers are better off because they buy more units at a lower price per unit. Sellers are better off because they sell more units and receive a higher price per unit. However, the subsidy payments exceed the additional benefits producers and consumers receive, and since the subsidy payments ultimately have to come from consumers and producers, society as a whole is worse off by area F; the deadweight loss of the subsidy is F.

Variable	Before	After	Change
Demanders' Price ($/unit)	P^*	P_D	$(P_D - P^*) < 0$
Sellers' Price ($/unit)	P^*	P_S	$(P_S - P^*) > 0$
Quantity Traded (units)	Q^*	Q^{sub}	$(Q^{sub} - Q^*) > 0$
Consumer Exp. ($)	$(P^* \cdot Q^*)$	$(P_D \cdot Q^{sub})$	$[(P_D \cdot Q^{sub}) - (P^* \cdot Q^*)] \gtreqless 0$
Producer Revenue ($)	$(P^* \cdot Q^*)$	$(P_S \cdot Q^{sub})$	$[(P_S \cdot Q^{sub}) - (P^* \cdot Q^*)] > 0$
Govt. Exp. ($)	0	$s \cdot Q^{sub}$	$s \cdot Q^{sub} > 0$
Consumer Surplus	$(A + B)$	$(A + B+$ $C + G + H)$	$(C + G + H) > 0$
Producer Surplus	$(C + D)$	$(B + C + D + E)$	$(B + E) > 0$
Gains from Trade	$(A + B+$ $C + D)$	$(A + 2B + 2C+$ $D + E + G + H$	$(B + C + E+$ $G + H) > 0$
Government Exp.	0	$(B + C + E+$ $F + G + H)$	$(B + C + E + 0$ $F + G + H) > 0$
Total Surplus	$(A + B+$ $C + D)$	$(A + B + C+$ $D - F)$	$-F < 0$

1.6 Welfare Analysis and International Trade

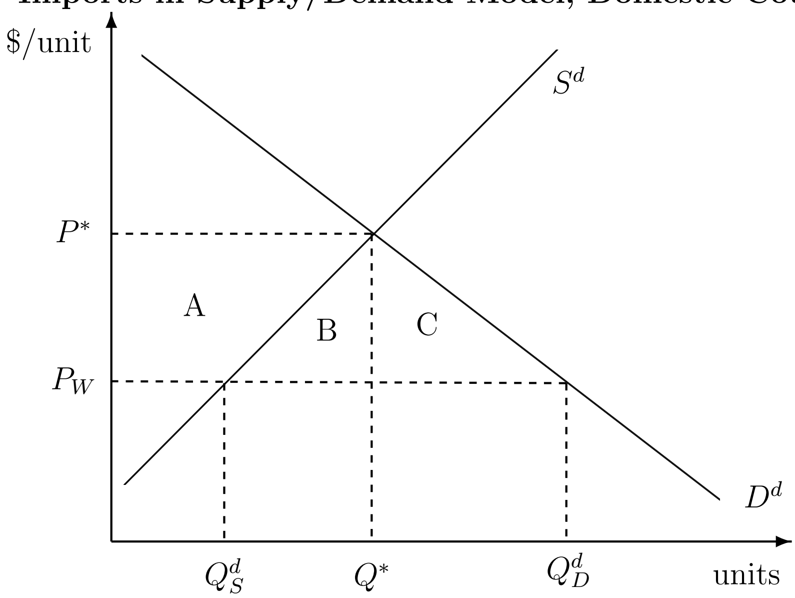
I now extend the basic supply and demand model to include trade with foreign countries. For this analysis, I will focus on what happens in the domestic country. A richer analysis would examine what happens in foreign countries as well.

1.6.1 The Basic Setup

Denote the price domestic demanders pay as P_D^d, the price domestic sellers receive as P_S^d. The demand function of domestic consumers is given by $Q_D^d(P_D^d)$, and the supply function of domestic producers is given by $Q_S^d(P_S^d)$.

In the absence of taxes or subsidies, the price buyers pay and the price sellers receive is the same, i.e., $P_S^d = P_D^d = P$. For the remainder of this analysis, I will assume that there are no taxes, subsidies, price controls, or quantity controls in the domestic market. In the standard supply and demand model, the equilibrium price was the one that equated quantity supplied and quantity demanded, i.e., $Q_S^d(P^*) = Q_D^d(P^*)$; the quantity traded is $Q^* = Q_S^d(P^*) = Q_D^d(P^*)$. This is what I will refer to as the **autarky outcome**, i.e., the equilibrium in the market when there is no international trade.

Figure 8: Imports in Supply/Demand Model, Domestic Country



Suppose the government opens up the market to free trade with other countries. Suppose further that the price in the world market P_W is less than the autarky price, i.e., $P_W < P^*$. If the quantity demanded at the world price is greater than what domestic producers will supply at the world price, i.e., $Q_D^d(P_W) > Q_S^d(P_W)$, domestic consumers will want to import units of the good. How much will they import? They will import the difference between the quantity demanded at the world price and the quantity they get from domestic producers at the world price; import demand is thus $MD = (Q_D^d(P_W) - Q_S^d(P_W))$.

If free trade is allowed, domestic consumer surplus increases by $(A + B + C)$; domestic consumers are better off because they buy more units and pay a lower price per unit. Domestic producer surplus falls by A; domestic producers are worse off because they are selling fewer units and receive a lower price per unit. However, the additional surplus received by domestic consumers is more than enough to compensate domestic producers for their loss. After doing so, total surplus in the domestic country increases by $(B + C)$. The domestic country is unambiguously better off with free trade.

Suppose instead the price in the world market is greater than the autarky price, i.e., $P_W > P^*$. If the quantity supplied at the world price is greater than what domestic consumers demand at the world price, i.e., $Q_D^d(P_W) < Q_S^d(P_W)$, domestic producers will want to export units of the good. How much will they want to export? They will want to export the difference between the quantity supplied at the world price and the quantity demanded by domestic consumers at the world price; export supply is thus thus $XS = (Q_S^d(P_W) - Q_D^d(P_W))$.

Similar analysis as to that done above can show that domestic consumers will be worse off because they buy fewer units and pay a higher price per unit. However, these losses will be outweighed by increase in domestic producer surplus (domestic producer sell more units at a higher price). Domestic consumers can be fully compensated from the gains to domestic producers, and there will still be surplus left. Total surplus in the domestic country rises due to trade. The domestic country is unambiguously better off with free trade.

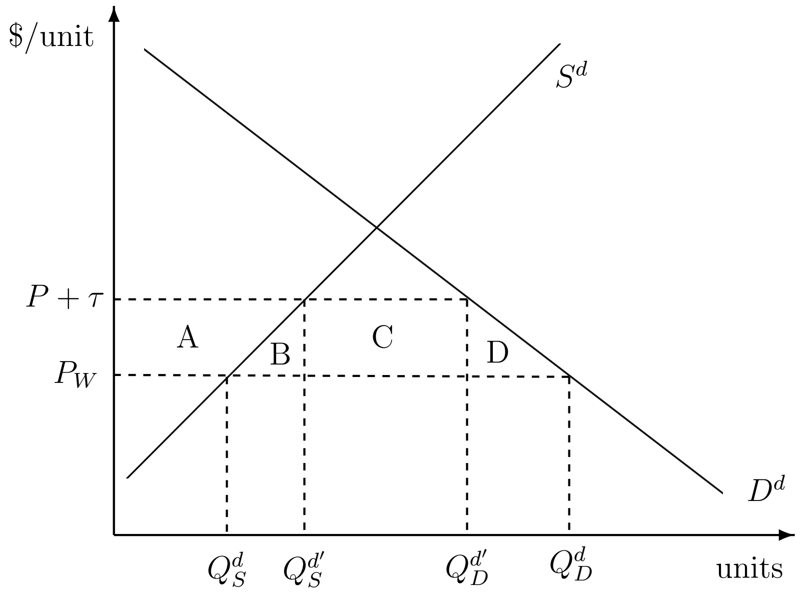
For the remainder of this section, I will analyze only the case where the domestic country imports goods from abroad. I will always start from the free-trade outcome, impose the government policy, and see how domestic consumer and producer surplus change as a result of the policy.

1.6.2 Tariff

Definition. **Tariff:** a tax on the quantity of goods that are imported.

I will focus on a per-unit tariff of $\$\tau$/unit. I will assume that the domestic country is "small" relative the size of the world market, so that the supply curve it faces in the world market is perfectly elastic. This implies the entire portion of the tariff is born by consumers in the domestic market. That is, the price consumers pay is the world price plus the per-unit tariff: $P_D^d = P_W + \tau$.

Figure 9: Import Tariff



Then, ΔTotal Surplus = ΔCS +ΔPS +ΔTariff Revenue. With upward-sloping supply and downward-sloping demand, domestic producers are unambiguously better off; their surplus increases by A. Domestic consumers are unambiguously worse off (since by assumption the price they pay rises by the entire amount of the tax); their surplus decreases by $(A+B+C+D)$. The gains from trade fall by $(B+C+D)$. Under the tariff, $(Q_D^{d'} - Q_S^{d'})$ are imported; tariff revenue corresponds to area C. Thus, total surplus in the domestic country falls by $(B+D)$; the deadweight loss is $(B+D)$. There is a deadweight loss because fewer units are being consumed in the domestic market, and the price domestic consumers pay exceeds the world price.

You may be concerned about the assumption of perfectly elastic supply in the world market. If the country is "large" relative the world market, it may be able to make foreign producers and consumers bear part of the tariff. Indeed, it is possible that imposing a tariff could raise total surplus in the domestic country. However, the long-term costs of imposing such a trade barrier could (and often do) easily and dramatically outweigh the short-term benefits. Economists generally advocate for reductions in trade barriers.

1.6.3 Import Quota

Definition. **Quota:** Situation where the government allows only a certain quantity of the good, say $\overline{Q}$, to be imported.

When the quota is less than the quantity that is imported at the world price, the quota is binding. Two cases need to be considered, one where domestic agents hold the quota and one where foreign agents hold the quota. I examine each in turn.

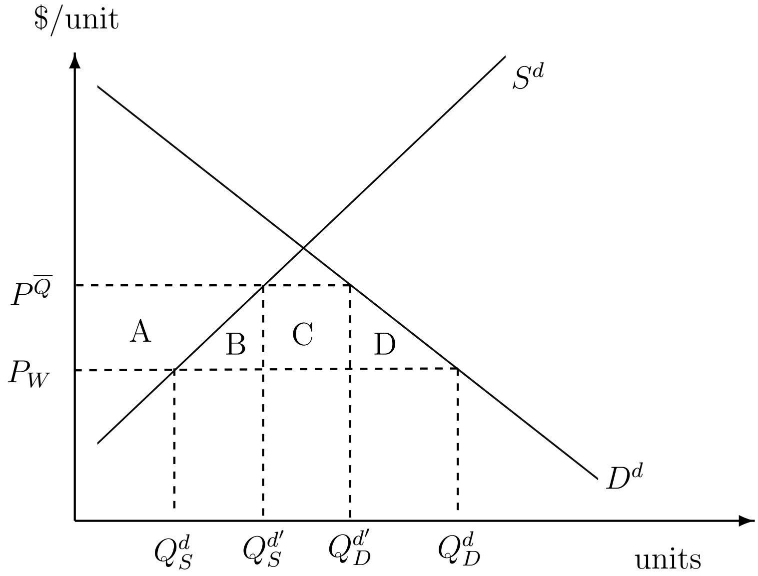
Case 1: Domestic Agents Hold Quota

In this case, you can think of it as if you're an importer of a good. The government has given you the right to import so many units of this product. You can buy units on the world

market at a price of P_W. Given that the quota is binding, consumers in the domestic country will bid up the price of the good to, say, $P^{\overline{Q}}$. Those who own the quota get rents; they buy at a lower price and sell at a higher price. The rents from the quota are $(P^{\overline{Q}} - P_W) \cdot \overline{Q}$, and these stay within the domestic country in this case.

Figure 10 shows the results of this policy. Domestic consumer surplus falls by $(A+B+C+D)$. Domestic producer surplus increases by A. The quota rents C accrue to domestic agents. So, the change in total surplus is $\Delta TS = -(A+B+C+D) + A + C = -(B+D)$. The deadweight loss from the import quota is $(B+D)$.

Figure 10: Import Quota



Case 2: Foreign Agents Hold Quota

In this case, you can think of it as the government has given to foreign producers the right to export so many units of this good to the domestic country. Foreign producers could sell the good in the world market and receive the world price P_W, or they could sell the good in the domestic country. As before, given that the quota is binding, consumers will bid up the price of the good to say $P^{\overline{Q}}$. This time, foreign producers get the rents from the quota $(P^{\overline{Q}} - P_W) \cdot \overline{Q}$. Since the rents do not accrue in the domestic country, they are not used to compute the change in total surplus.

Again, I refer the reader to Figure 10. Domestic consumer surplus falls by $(A+B+C+D)$; domestic producer surplus increases by A. Since foreign agents hold the quota, the rents go to foreigners. So, the change in total surplus is $\Delta TS = -(A+B+C+D)+A = -(B+C+D)$. The deadweight loss from the import quota is $(B+C+D)$. There is a deadweight loss associated with this policy, and it is larger than the deadweight loss was when domestic agents hold the quota.

1.6.4 Voluntary Export Restraint

Definition. **Voluntary Export Reduction/Restraint:** situation where a foreign country imposes a restriction on the quantity of exports sent from the foreign country to the domestic country.

The results are qualitatively similar to those of a quota held by foreign agents since foreign agents are voluntarily restricting the amount they export to the domestic country. This increases the price they receive. Thus, they get some rents from this activity.

2 Technology, Costs of Production, and The Firm's Problem

2.1 Technology

A firm's technology describes how inputs are transformed into output. When the firm produces one good, the firm's technology can be described by a production function. For simplicity, let's consider only two inputs, capital and labor. Let K be the amount of capital (machines) used, L be the amount of labor used. The firm's output, Q, is described by the production function $Q = f(K, L)$.

Definition. **Short-run:** period of time in which at least one factor of production is fixed.

- Generally, we think of capital as fixed in the short-run, i.e., $K = \overline{K}$. So, the firm's production function becomes $Q = f(\overline{K}, L)$. The firm can vary only the amount of labor it uses in the short-run.

Definition. **Long-run:** period of time in which all factors can be varied.

2.1.1 Properties of the Production Function

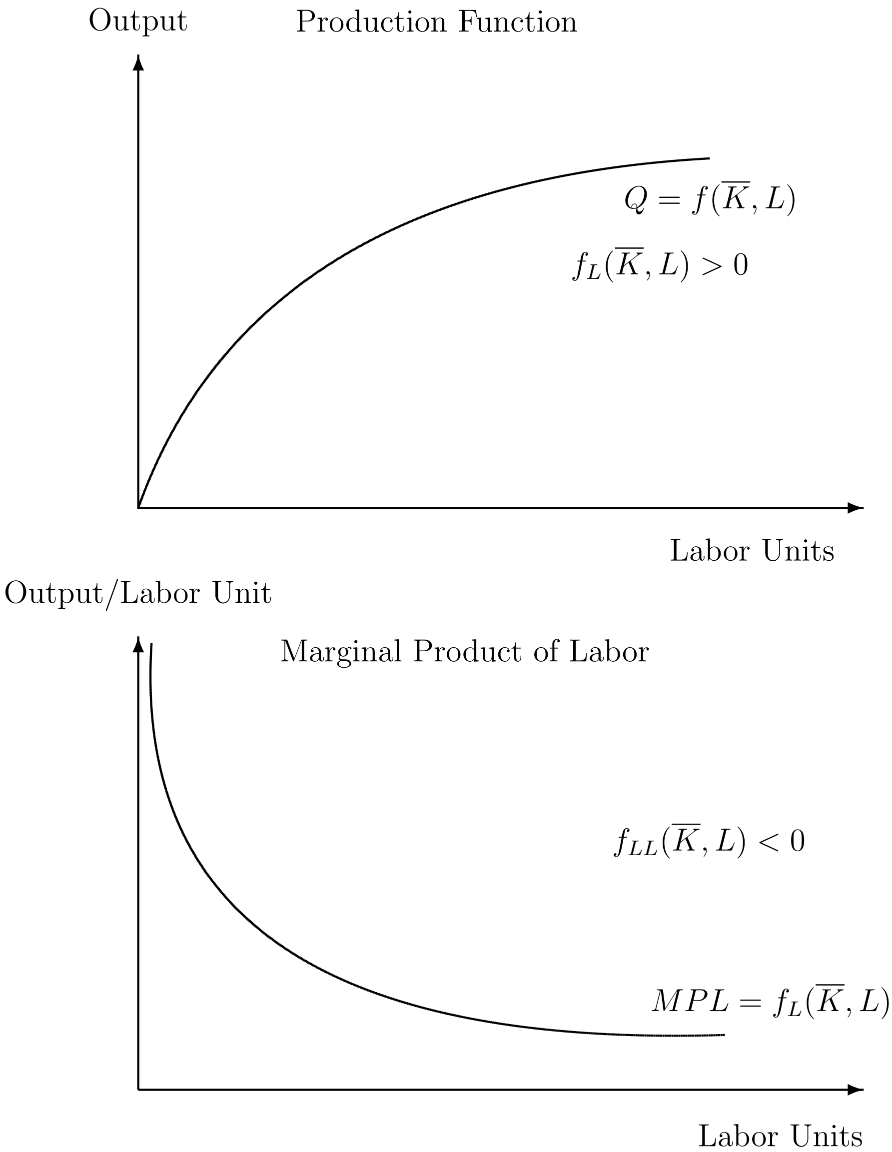
Definition. **Marginal Product:** extra output we get as we change a variable factor of production, say X, holding all other factors constant: $MPX = \frac{\Delta Q}{\Delta X}$.

- With inputs capital and labor and with capital fixed in the short-run, we can look at the marginal product of labor: $MPL = \frac{\Delta Q}{\Delta L}$, which is the extra output we get as we change the amount of labor used holding capital fixed.
  - If the firm can choose fractional amounts of labor, then the marginal product of labor is the partial derivative with respect to labor of the production function: $MPL = \frac{\partial Q}{\partial L} = f_L(K, L)$.
- The marginal product of capital is $MPK = \frac{\Delta Q}{\Delta K}$, which tells us how much extra output we get as we change the amount of capital used holding labor fixed.
  - If the firm can choose fractional amounts of capital, then the marginal product of capital is the partial derivative with respect to capital of the production function: $MPK = \frac{\partial Q}{\partial K} = f_K(K, L)$.

Definition. **Diminishing Marginal Returns:** This is a possible property of the production function. It says that as we increase the use of a variable input, holding all other factors fixed, the **extra** amount of output we get eventually declines. This is akin to diminishing marginal utility from the consumer's problem.

- Diminishing marginal returns is a second derivative thing. Let the variable factor be X. Diminishing marginal product is equivalent to $\frac{\partial^2 Q}{\partial X^2} = f_{XX} < 0$.

Figure 11: Diminishing Marginal Product



Definition. **Average Product:** output per unit of input: $APX = \frac{Q}{X}$.

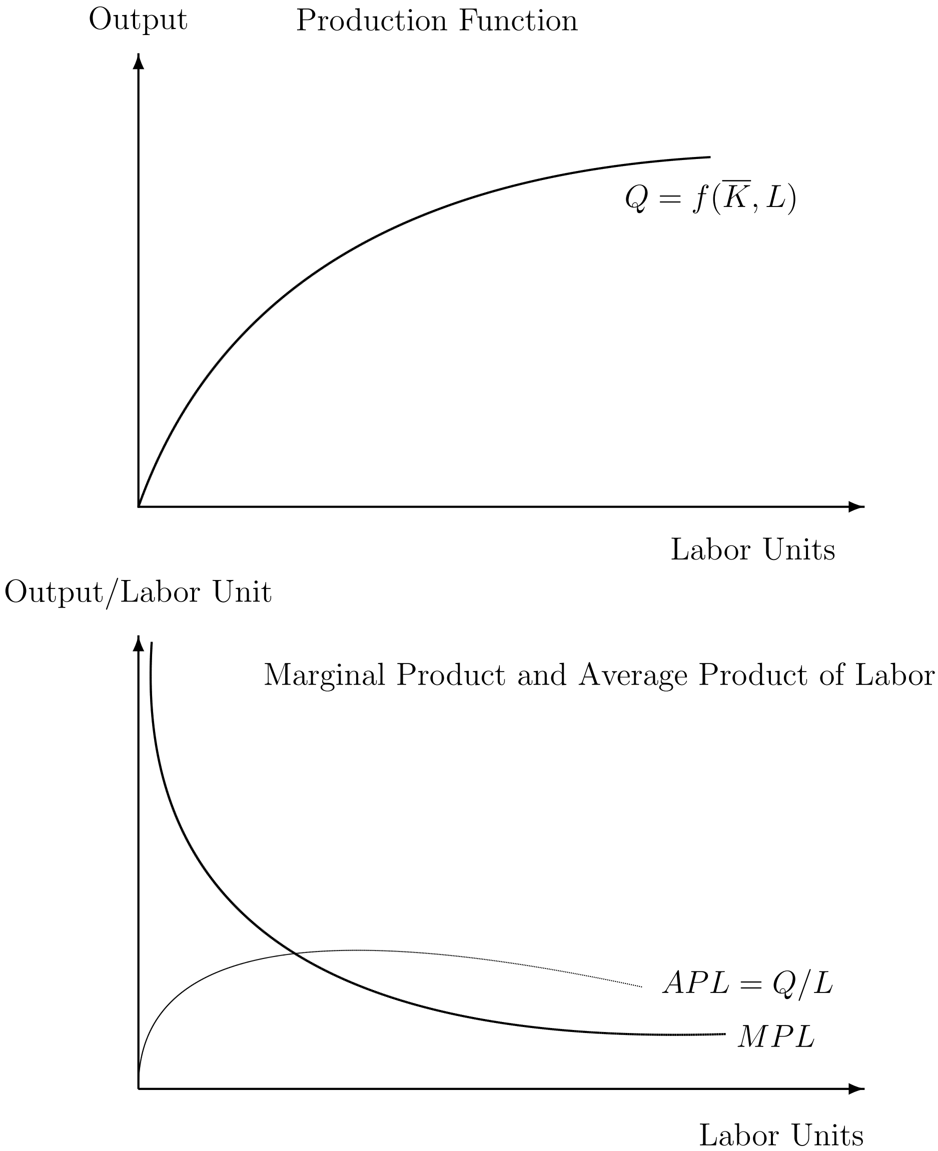
- The average product of labor is output per unit of labor: $APL = \frac{Q}{L}$.
- Similarly, the average product of capital is $APK = \frac{Q}{K}$.

Relationship between Average Product and Marginal Product

- When the marginal is above the average, it brings the average up; when the marginal is below the average, it brings the average down.
- The marginal product intersects the average product at the average product's maximum.

This is just like with exam scores. If your next exam score is greater than your average, it brings your average exam score up; if your next exam score is lower, it brings your average exam score down; and if your next exam score is the same as your average exam score, your average score doesn't change.

Figure 12: Marginals vs. Averages



Internal Economies of Scale
This is a long-run concept within the firm. It tells us how output changes if we multiply all factors of production by the same amount.

- **Increasing Returns to Scale (IRTS):** The production function exhibits IRTS if we multiply all factors of production by λ and we get more than λ times the original amount of output. For example, we would have IRTS if we doubled all inputs and got three times as much output.
  - A production function exhibits increasing returns to scale if it is homogeneous of degree $t > 1$. Remember, a function is said to be homogeneous of degree t if, for all $\lambda > 0$, $f(\lambda K, \lambda L) = \lambda^t f(K, L)$.
- **Constant Returns to Scale (CRTS):** The production function exhibits CRTS if we multiply all factors of production by λ and we get exactly λ times the original amount of output. For example, we would have CRTS if we doubled all inputs and got double the output.
  - A production function exhibits constant returns to scale if it is homogeneous of degree $t = 1$.
- **Decreasing Returns to Scale (DRTS):** The production function exhibits DRTS if we multiply all factors of production by λ and we get less than λ times the original amount of output. For example, we would have DRTS if we doubled all inputs and got 1.5 times the output.
  - A production function exhibits decreasing returns to scale if it is homogeneous of degree $t < 1$.

2.2 Costs of Production

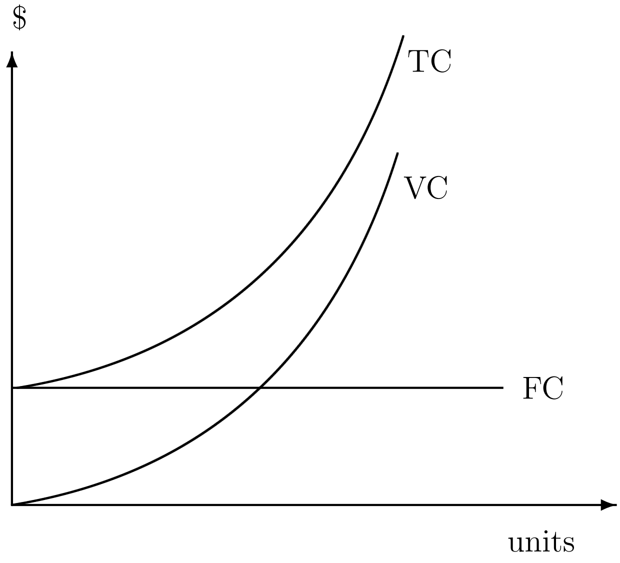
A firm's technology implies its costs. I've already introduced the notation for what follows in section 1.1, but I will reintroduce it. I will assume that the price of the firm's inputs are fixed and constant, i.e., the firm has no power in determining the price of its inputs.

Definition. **Fixed Cost (FC):** Those costs that do not vary with output.

Definition. **Variable Cost (VC):** Those costs that vary with output: $VC = VC(Q)$.

Definition. **Total Cost (TC):** The sum of fixed and variable costs: $TC(Q) = FC + VC(Q)$.

Figure 13: Fixed, Variable, and Total Costs



Definition. **Marginal Cost (MC):** The extra cost associated with producing an extra unit of the good: $MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta VC}{\Delta Q}$ since $\frac{\Delta FC}{\Delta Q} = 0$ by definition.

The variable cost a producer incurs from producing the Q units is the sum of the marginal costs: $VC = \sum_{i=1}^{Q} MC_i$.

Definition. **Average Total Cost (ATC):** cost per unit of output: $ATC = \frac{TC}{Q}$.

Definition. **Average Fixed Cost (AFC):** fixed cost per unit of output: $AFC = \frac{FC}{Q}$.

- Since, by definition, fixed costs don't vary with output, average fixed cost falls continuously as output rises, i.e., $\frac{dAFC}{dQ} = \frac{d}{dQ}(\frac{FC}{Q}) = -\frac{FC}{Q^2} < 0$.

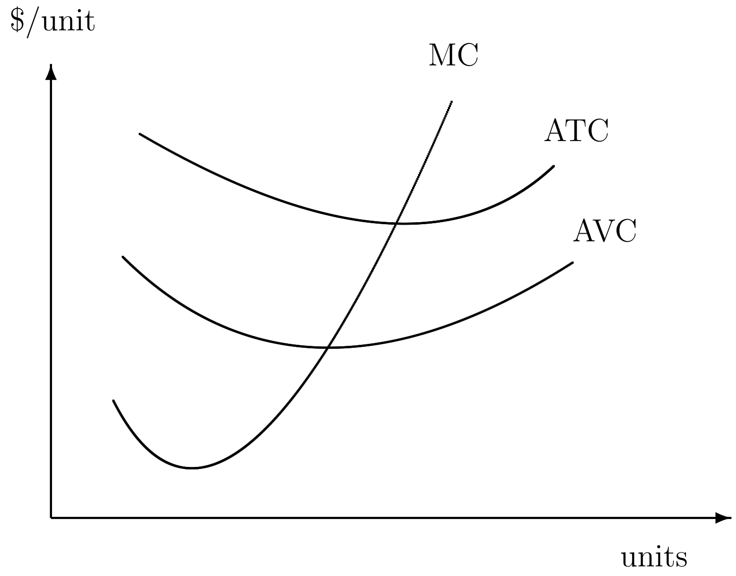
Definition. **Average Variable Cost (AVC):** variable cost per unit of output: $AVC = \frac{VC}{Q}$.

Since $TC = FC + VC$, dividing both sides by $Q > 0$, it follows that $ATC = AFC + AVC$.

Relationship between Marginal Cost and Average Cost

If MC exceeds AVC, the AVC is rising; similarly for ATC. Likewise, if MC is less than AVC, the AVC is falling; similarly for ATC. Thus, MC intersects AVC and ATC at the minimum of AVC and at the minimum of ATC.

Figure 14: U-Shaped Costs



For those who might be confused by intuition and want math to clear things up, I'll prove what I've said above. Recall, that total cost is a function of quantity, i.e., $TC = TC(Q)$, and $MC = TC'(Q) = VC'(Q)$. Therefore, $\frac{dATC}{dQ} = \frac{d}{dQ}(\frac{TC}{Q}) = \frac{Q \cdot TC' - TC \cdot 1}{Q^2} = \frac{Q \cdot MC - TC}{Q^2} = \frac{MC}{Q} - \frac{1}{Q}\frac{TC}{Q} = \frac{1}{Q}(MC - ATC)$. Thus, for all $Q > 0$, $\frac{dATC}{dQ} > 0$ only if $MC > ATC$, $\frac{dATC}{dQ} = 0$ when $MC = ATC$, and $\frac{dATC}{dQ} < 0$ when $MC < ATC$.

- For those who are anal about second-order conditions, we need $\frac{d^2ATC}{dQ^2} > 0$ for ATC to be minimized. So, by the product rule and some rearranging, we get $\frac{d^2ATC}{dQ^2} = \frac{d}{dQ}\frac{1}{Q}(MC - ATC) = \frac{MC'}{Q} - \frac{2}{Q^2}(MC - ATC)$. Since, at the point where $\frac{dATC}{dQ} = 0$, $MC \equiv ATC$, we have $\frac{d^2ATC}{dQ^2} = \frac{MC'}{Q}$. So, for $Q > 0$, we need $MC' > 0$, i.e., marginal cost to be upward-sloping, for ATC to be minimized at the quantity where $MC = ATC$.

Similarly, $\frac{dAVC}{dQ} = \frac{1}{Q}(MC - AVC)$. Thus, for all $Q > 0$, $\frac{dAVC}{dQ} > 0$ only if $MC > AVC$, $\frac{dAVC}{dQ} = 0$ when $MC = AVC$, and $\frac{dAVC}{dQ} < 0$ when $MC < AVC$. For AVC to be minimized where $MC = AVC$, the second-order condition again requires the marginal cost curve to be upward-sloping.

Consider a case where capital is fixed in the short-run, i.e., $K = \overline{K}$. Suppose we can rent units of capital for a fixed rate of $\$R$/unit. Suppose also that labor L is the only variable factor of production, and we can hire labor at a fixed wage of $\$w$/labor unit.

- **Short-run**
  - The firm's costs are $TC = R\overline{K} + wL$. Fixed costs are $R\overline{K}$, and variable costs are wL since capital is fixed in the short-run but labor is variable.
  - Next, average fixed cost is $AFC = \frac{FC}{Q} = \frac{R\overline{K}}{Q} = \frac{R}{APK}$, where $APK = \frac{Q}{K}$ is the average product of capital. Similarly, average variable cost is $AVC = \frac{VC}{Q} = \frac{wL}{Q} = \frac{w}{APL}$, where $APL = \frac{Q}{L}$ is the average product of labor. This implies as the average

product of labor rises (falls), average variable cost and therefore average total cost falls (rises).

- Next, marginal cost is $MC = \frac{\Delta wL}{\Delta Q} = \frac{w\Delta L}{\Delta Q} = \frac{w}{MPL}$, where $MPL = \frac{\Delta Q}{\Delta L}$ is the marginal product of labor. As the marginal product of labor rises (falls), the marginal cost falls (rises).
  * So, from our requirement above that marginal cost be upward-sloping at the points of minimum AVC and minimum ATC, we need MPL to be downward-sloping, i.e., falling, at the quantity where $MC = AVC$ and where $MC = ATC$.

- **Long-run**
  - In the long-run, the firm's costs are $TC = RK + wL$. All costs are variable in the long-run by definition (neither capital nor labor is fixed).
  - Average total cost in the long-run is $ATC = \frac{RK}{Q} + \frac{wL}{Q} = \frac{R}{APK} + \frac{w}{APL}$.
  - Marginal cost in the long-run is $MC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta(RK+wL)}{\Delta Q} = \frac{R}{MPK} + \frac{w}{MPL}$.

2.3 Profit Maximization

The fundamental assumption of producer theory is that firms maximize their profits subject to their technology. That is, they choose their inputs (which by the firm's technology transforms them into output) to maximize profits.

For this class, I will only consider the case where the firm produces one good. Let the firm's total revenue and total cost, as a function of the quantity it produces, be denoted as $TR(Q)$ and $TC(Q)$, respectively. The firm's profit as a function of quantity, denoted as $\pi(Q)$, is its revenue less its costs: $\pi(Q) = TR(Q) - TC(Q)$. Let the firm's set of inputs be $\{X_1, ..., X_n\}$ and the firm's production function be $Q = f(X_1, ..., X_n)$. Then, formally, the firm's problem is written $Max_{\{X_1,...,X_n\}}\pi(Q) = TR(Q) - TC(Q)$ subject to $Q = f(X_1, ..., X_n)$.

How do we solve this problem? Let's first abstract from choosing the inputs and instead focus on choosing the level of output. It should be clear, though, that the choice of output leads directly to the choice of which inputs to use of how much of them to use.

- If we change output by a little bit, say ΔQ, the change in profit given this change in output is $\frac{\Delta\pi(Q)}{\Delta Q} = \frac{\Delta TR(Q)}{\Delta Q} - \frac{\Delta TC(Q)}{\Delta Q}$. We know from above, though, that $\frac{\Delta TR(Q)}{\Delta Q} = MR$ and $\frac{\Delta TC(Q)}{\Delta Q} = MC$. Thus, the change in profit given a change in output is equal to marginal revenue minus marginal cost, which is the producer surplus on a unit of output! **On the last unit produced**, we want $\frac{\Delta\pi(Q)}{\Delta Q} \geq 0 \Leftrightarrow MR \geq MC$.
  - **If we can choose very small amounts of output, on the last unit we produce, we want $MR = MC$.**

- We produce a unit of output so long as the producer surplus on that unit is weakly positive: $MR \geq MC$. Indeed, **if** $MR > MC$, we need to **increase output** to maximize profits.

- If, on the last unit we produce, $MR < MC$, we're losing money on that unit, so we shouldn't produce it; i.e., **if** $MR < MC$, we need to **produce less**.

If the firm produces, its profits are $\pi = TR - (FC + VC) = TR - FC - VC = TR - VC - FC = PS - FC$, where PS is producer surplus. Notice that if the firm doesn't produce, its profits are $\pi = -FC$ since it has neither revenue nor variable costs. So, we can see that producer surplus is the contribution to profits from actually producing. Maximizing profits necessarily requires maximizing producer surplus!

Now, let's take a closer look at the problem where the firm explicitly chooses inputs to maximize profits subject to its technology. To make things a little more tractable, let's assume the firm takes prices in output and input markets as fixed and constant. In particular, suppose the price the firm receives is P_S, measured in dollars per unit of output; and let the cost of input X_i be w_i, measured in dollars per unit, for $i = 1, ..., n$.

The firm's problem now is $Max_{\{X_1,...,X_n\}} \pi(Q) = P_S Q - (w_1 X_1 + ... + w_n X_n)$ subject to $Q = f(X_1, ..., X_n)$. Substituting in the production function, the firm's profit is $\pi = P_S f(X_1, ..., X_n) - (w_1 X_1 + ... + w_n X_n)$.

If we hire a little bit more X_i, the change in profit given a change in X_i is $\frac{\Delta \pi}{\Delta X_i} = P_S \frac{\Delta Q}{\Delta X_i} - w_i$. The first term is called the **marginal revenue product of** X_i. It tells us the **extra** revenue the firm gets from hiring a little more X_i. Notice that $\frac{\Delta Q}{\Delta X_i}$ is the marginal product of X_i! So, the marginal revenue product, given our assumptions of fixed prices, is the price the firm receives times the marginal product of X_i. The second term, w_i, is the marginal cost of hiring a little bit more X_i. The firm should hire X_i so long as $P_S \cdot MPX_i \geq w_i$.

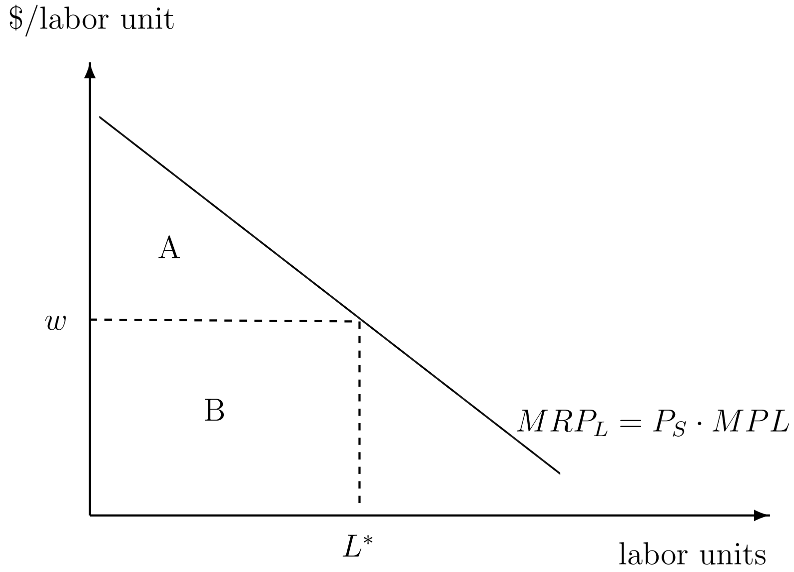
- The firm should hire more X_i if $P_S \cdot MPX_i > w_i$.

- The firm should hire less X_i if $P_S \cdot MPX_i < w_i$.

- The firm is maximizing profits if, **for all** X_i, $P_S \cdot MPX_i = w_i$.

To make this a little more concrete, suppose the firm uses two inputs, labor L and capital K. Suppose further the firm is in the short-run so that $K = \overline{K}$. So, the firm's problem is to choose labor to maximize profits. Let the wage rate be \$$w$/labor unit. The marginal revenue product of labor is the additional output the firm gets from hiring another unit of labor, the marginal product of labor, times the price it receives for selling each of these units: $MRP = P_S \cdot MPL$. The marginal cost of hiring another unit of labor is w. So, on the last unit of labor, we want the marginal revenue product of labor equal to the marginal cost of labor (the wage rate): $P_S \cdot MPL = w$.

Figure 15: Marginal Revenue Product, Input Choice, Producer Surplus



In Figure 15, the wage rate is w. The firm's marginal revenue product of labor curve, MRP_L, is the firm's demand curve for labor. Given these, the firm hires L^* units of labor at the market wage rate. The total benefit the firm gets from hiring these units is $(A + B)$, which is the firm's total revenue. The (variable) cost associated with hiring L^* units of labor is given by B. Thus, the firm's surplus from hiring these units is A.

If we were in the long-run and thus could choose capital as well, we'd have the following two conditions to satisfy: $P_S \cdot MPL = w$ and $P_S \cdot MPK = R$, where R is the cost of a unit of capital. Since these two things are equal, we can divide each side: $\frac{P_S \cdot MPL}{P_S \cdot MPK} = \frac{w}{R}$. The left-hand side is called the **marginal rate of technical substitution**, and the right-hand side is the **economic rate of substitution** for the firm. We can rearrange once more and get $\frac{MPL}{w} = \frac{MPK}{R}$; this looks suspiciously like what we came up with for consumers!

- Hire more labor and less capital if $\frac{MPL}{w} > \frac{MPK}{R}$.
- Hire less labor and more capital if $\frac{MPL}{w} < \frac{MPK}{R}$.
- Profits are maximized if, on the last unit of labor and capital, $\frac{MPL}{w} = \frac{MPK}{R}$.

Long-story short: choose quantity to make marginal revenue equal marginal cost on the last unit produced. If you can choose your inputs, choose them to make the marginal revenue product of that input equal to its marginal cost; do so for all your inputs.

3 Perfect Competition

In this section, I develop the main points you should know from producer theory in a perfectly competitive market. First, I outline the assumptions. Then I develop some results for the firm operating in the short-run and then in the long-run.

3.1 Assumptions

Consider a market for a good. Assume:

1. There are lots and lots and lots of firms. Each firm's production is small relative the size of the market.
2. Each firm produces a homogeneous good. That is, consumers cannot tell one firm's product from another (think agricultural goods... I can't tell Farmer A's corn from Farmer B's corn).
3. Each firm takes the price of the product sold in the market as a given and the prices of its inputs as given. The firm's choice of output has no impact on the price of the good sold and no impact of the prices of its inputs. The firm can sell as many units of the good it wants at the price given in the market.
4. Firms choose output to maximize profits.
5. Firms can freely and costlessly enter and exit this market any time they want.

Later on in the course, we'll relax these assumptions and see how the conclusions change.

3.2 Short-run Analysis

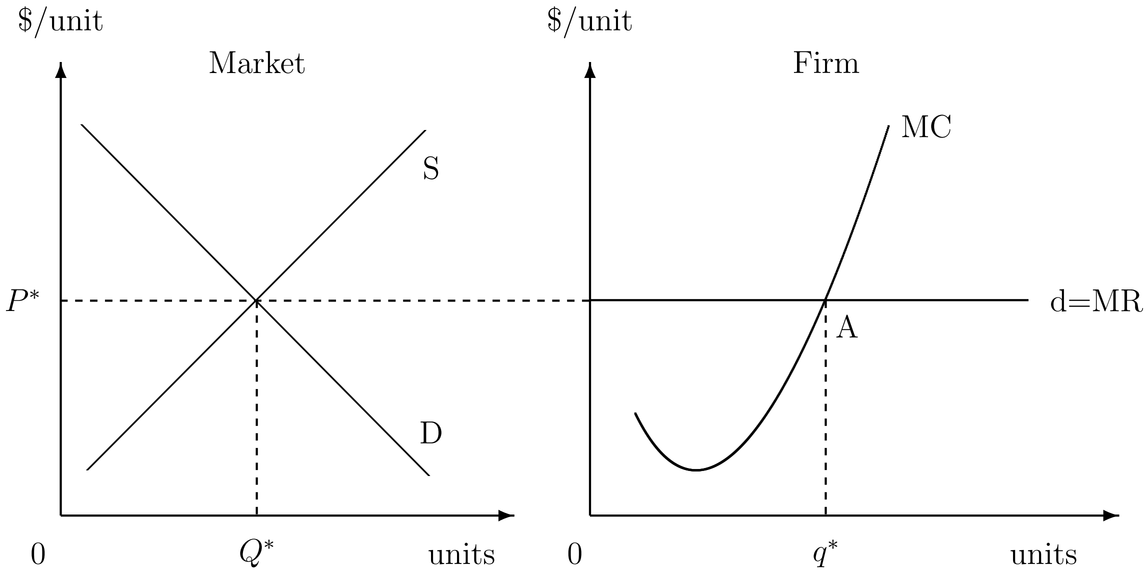
In the short-run, at least one of the firm's factors of production is fixed. For example, the number of ovens at Pizza House is fixed today. It would take time for them to order another oven, have it delivered, install it, and start baking pizzas in it.

From A3, the firm can sell as many units of the good it wants at the price that is given in the market. This means that the firm's demand curve is horizontal at the market price, i.e., perfectly elastic. Suppose the market price is \$10 per unit. The firm's demand curve is always its average revenue curve (it's the price per unit it can sell its good, \$10 per unit for each and every unit it sells). In the case of a perfectly competitive market, the firm's demand curve is also its marginal revenue curve. Each additional unit the firm sells bring in \$10 per unit; so the additional (marginal) revenue it gets from selling one more unit is \$10, which is just the price of the good. Now, A4 requires that the firm chooses the quantity it sells to be such that the marginal revenue from selling that last unit equals the marginal cost of selling that last unit.

With some slight abuse of notation from what I've done above, I'm going to let the quantity traded in the market be denoted by Q^*, and denote the quantity the firm chooses as q.

Consider Figure 16 below. The firm takes the equilibrium price in the market P^* as a given. It chooses the quantity where marginal revenue equals marginal cost q^*. Its revenue is the price it gets for each unit multiplied by the number of units it supplies to the market $TR = P^* \cdot q^*$. Graphically, this area corresponds to the rectangle $0P^*Aq^*$.

Figure 16: The Firm's Profit-Maximizing Quantity

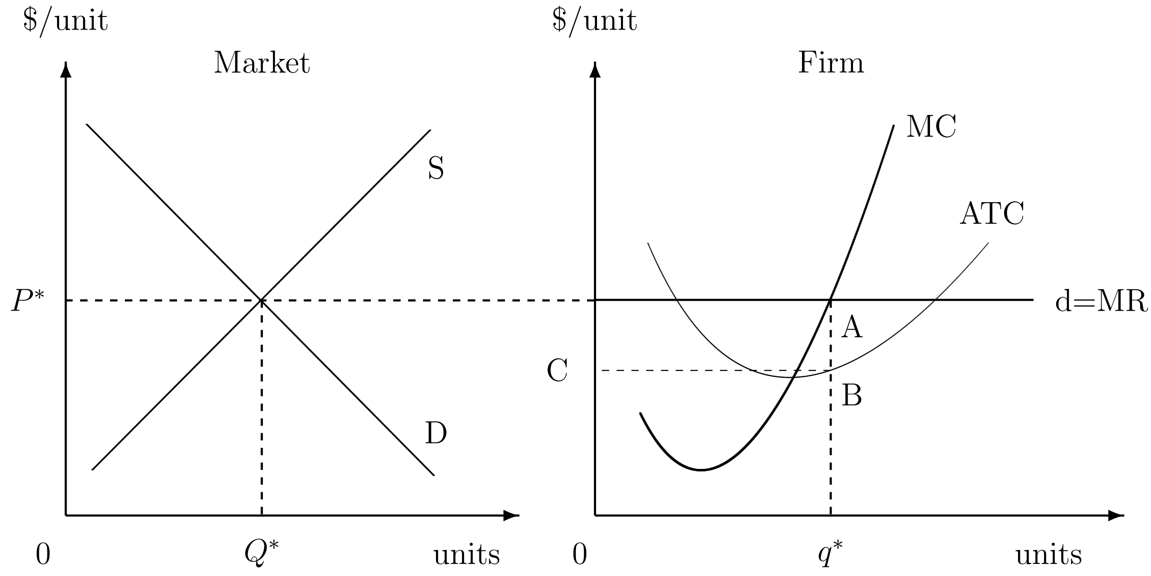


Next, we should ask ourselves what the firm's costs are. Remember that on our graph, the vertical axis is measured in dollars **per unit of output**. So, we can't graph total cost on the above graph. What we want to do is put the firm's cost **per unit of output** on the graph; this is just the average total cost curve.

In Figure 17, I've plotted the firm's average total cost curve, denoted ATC. Notice that when marginal cost exceeds average total cost, ATC is rising and when marginal cost is less than average total cost, ATC is falling. Thus, the marginal cost curve intersects the average total cost curve when average total costs are at their minimum.

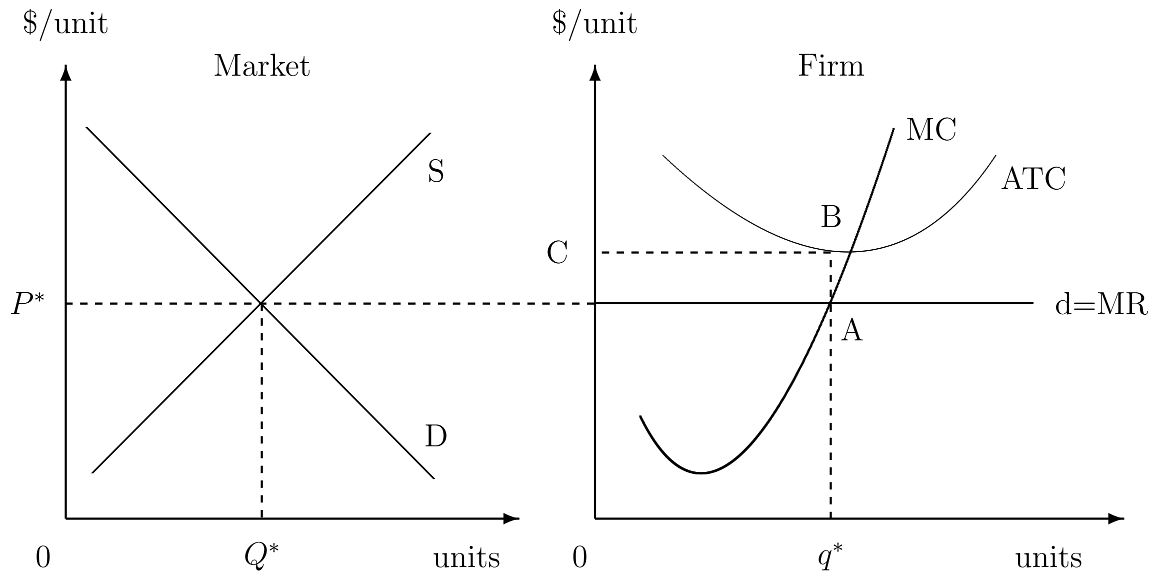
We figure out what the firm's cost per unit from producing q^* units is by drawing a line up from q^* to the ATC curve and looking at the dollar amount on the vertical axis. In this case, it costs the firm $\$C$/unit to produce q^* units. So, its total costs are $TC = C \cdot q^*$. Graphically, this corresponds to the rectangle $OCBq^*$. Now that we know both the firm's revenue and the firm's costs, we can determine its profit, denoted π. Profit is just revenue minus costs: $\pi = TR(q) - TC(q)$. In this case, profits are $\pi = P^* \cdot q^* - C \cdot q^* = (P^* - C) \cdot q^*$. Notice, this is just the revenue it got per unit P^* minus its cost per unit C multiplied by the number of units it sold q^*, i.e., $\pi = (P^* - ATC) \cdot q^*$. Graphically, the firm's profit corresponds to the rectangle P^*ABC. Notice the firm is earning a positive economic profit. This need not always be the case.

Figure 17: Firm's Revenue, Costs, and (Positive) Profit



Short-run Losses: Produce or Not?
Figure 18 illustrates a case where the firm is earning a negative profit (you could call it losses if you prefer) in the short-run. Revenues correspond to the rectangle $0P^*Aq^*$; costs correspond to the rectangle $0CBq^*$; the firm's profit (loss in this case) correspond to the rectangle P^*ABC.

Figure 18: Firm's Revenue, Costs, and (Negative) Profit



Given that the firm is earning a negative profit, we should ask ourselves, should this firm keep producing, or should it just shut down? To answer this question, we need to talk about the firm's fixed costs and variable costs.

Recall that the firm's variable costs are those that vary with the quantity produced; fixed

costs do not vary with the quantity produced. So if the firm produces, it must pay both its fixed costs and its variable costs. If, however, it decides to shut-down, it will only have to pay its fixed costs. So, the firm's decision to shut down hinges on one thing: are its losses from producing less than its fixed costs? If this is the case, the firm should produce in the short-run. However, if the firm's fixed costs are less than its losses from producing, then it should shut down.

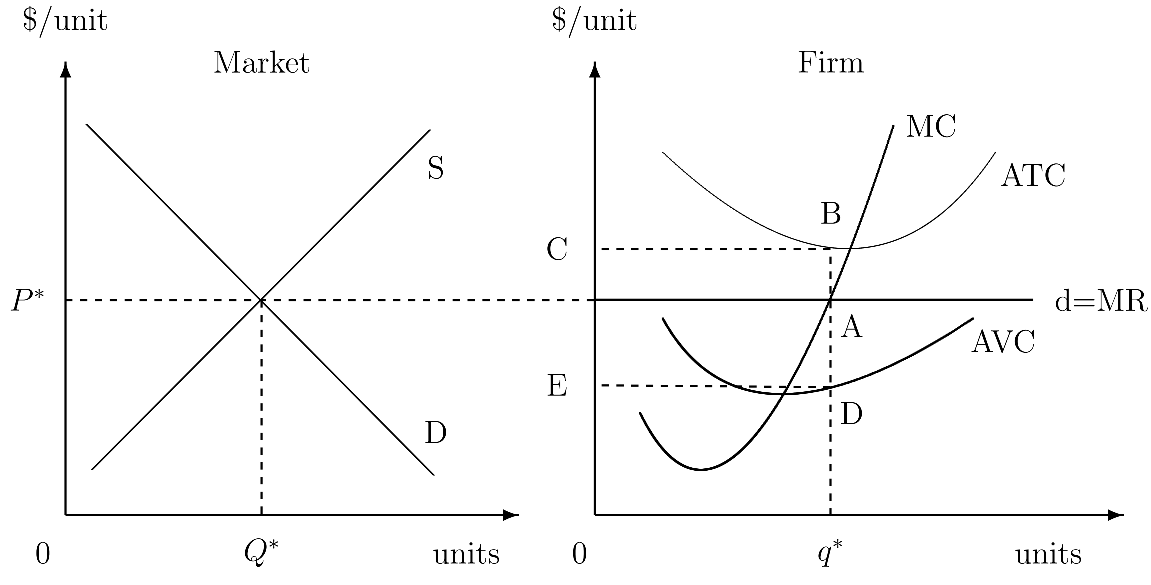
Case 1: $P^* > AVC \Leftrightarrow TR > VC \Leftrightarrow PS > 0$

Consider Figure 19. Again, notice the firm is losing money; its losses are denoted by the rectangle P^*ABC. To figure out the firm's variable costs, we again go up from q^* to the AVC curve, then see what the corresponding dollar amount is. In this case, the average variable cost of producing q^* units is $\$E$/unit. Variable costs then are $VC = \$E \cdot q^*$; this corresponds to the rectangle $0EDq^*$.

What are the firm's fixed costs? Well, we know that total costs are the sum of fixed costs and variable costs, i.e., $TC(q) = FC + VC(q)$. Thus, the firm's fixed costs are its total costs minus its variable costs $FC = TC(q) - VC(q)$. On the graph, the firm's total costs correspond to the rectangle $0CBq^*$; thus, its fixed costs correspond to the rectangle $CBDE$. Notice in this case its fixed costs exceed its losses if it produces. **Thus, when $P^* > AVC$, the firm should produce in the short-run.**

The intuition behind this is that it can take its revenue, pay all of its variable costs of production, and then take what's left (its producer surplus) and pay down its fixed costs by some amount. If it shuts down, it will lose the dollar amount corresponding to its fixed costs. So, it pays to produce here; the losses aren't quite as bad when the firm produces.

Figure 19: Short-run Losses but Produce in Short-run

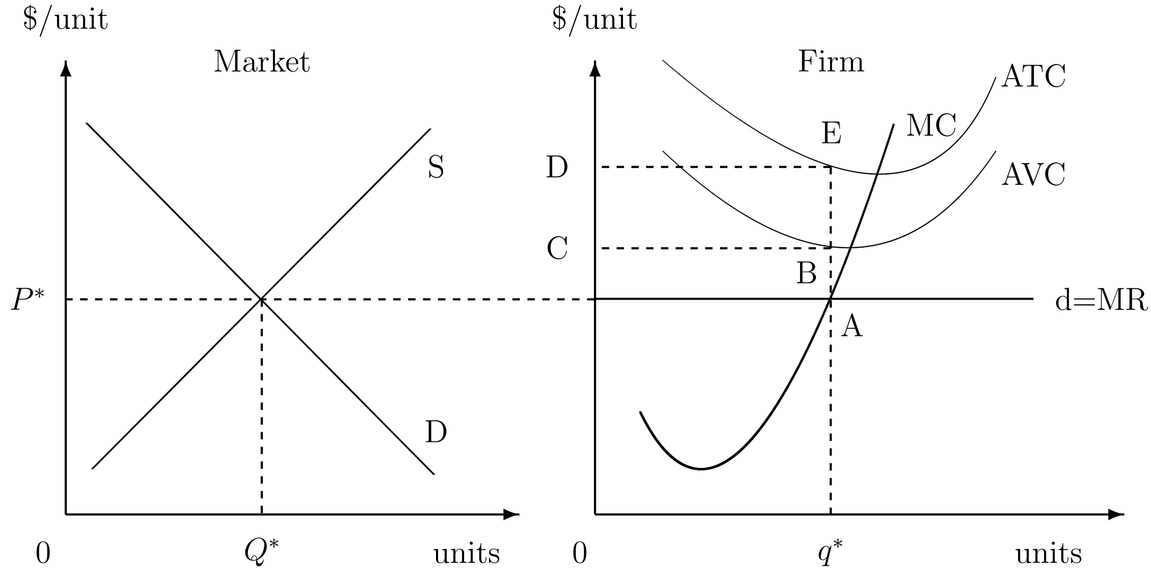


Case 2: $P^* < AVC \Leftrightarrow TR < VC \Leftrightarrow PS < 0$

Consider Figure 20. Again, notice the firm is losing money; its losses are denoted by the rectangle P^*AED. Its variable costs correspond to the rectangle $0CBq^*$, and its fixed costs are $BCDE$. In this case, its fixed costs are less than the losses it would incur if it produced. So, the firm should just shut down; its loss will be equal to its fixed costs. **Thus, when $P^* < AVC$, the firm should shut down in the short-run.**

Unlike in Case 1, the firm can't even pay its variable costs of production (its producer surplus is negative). If it shuts down, it just pays its fixed costs; if it produces, it loses its fixed costs as well as part of its variable costs. So, it should shut down and pay the fixed costs.

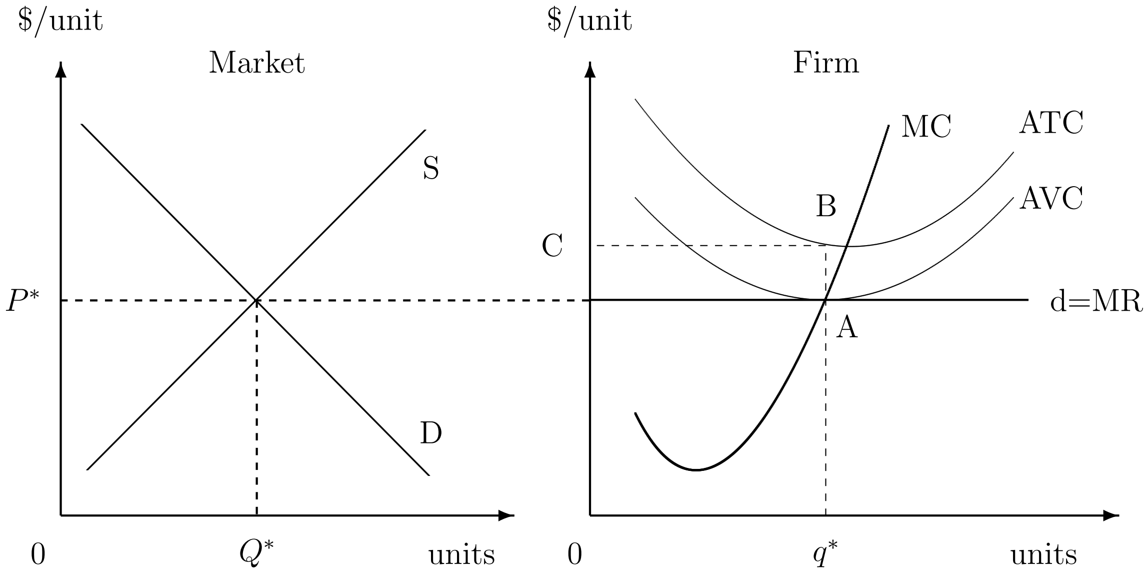
Figure 20: Short-run Losses and Shuts Down



Case 3: $P^* = AVC \Leftrightarrow TR = VC \Leftrightarrow PS = 0$

Consider Figure 21. Here, the minimum of the average variable cost curve is tangent to the firm's demand curve; the price the firm receives equals the minimum of its average variable costs. Again, notice the firm is losing money; its losses are denoted by the rectangle P^*ABC. In this case, its variable costs are just equal to its revenue (producer surplus is zero); they correspond to the rectangle $0P^*Aq^*$. Its fixed costs are equal to the losses it would incur if it produces; they correspond to the rectangle P^*ABC. So, the firm is indifferent to producing in the short-run and shutting down. We'll just assume that the firm goes ahead and produces in the short-run.

Figure 21: Short-run Losses, Shut Down or Produce

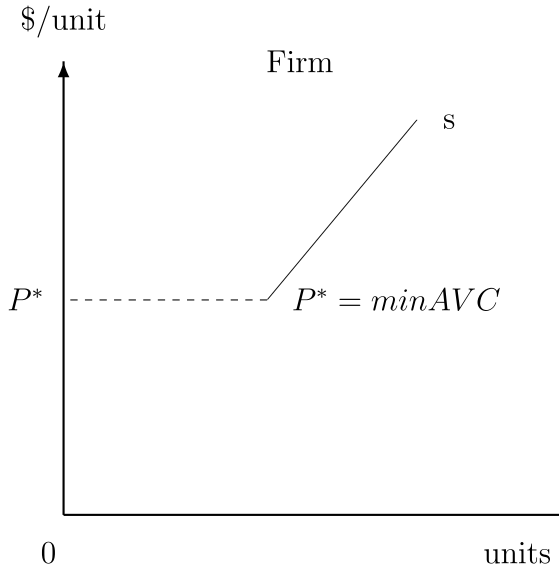


Shut-down Rule: The above analysis shows us that in the short-run, the firm should produce only if the price it receives is at least equal to the minimum average variable cost.

The Firm's Short-run Supply Curve
When the firm produces in the short run, the quantity it produces is found by equating the firm's marginal revenue and marginal cost. So, the firm's supply curve is the part of its marginal cost curve where the price is greater than the minimum of average variable cost:

$$q = \begin{cases} 0, & P^* < minAVC; \\ \text{q such that MR=MC}, & P^* \geq minAVC. \end{cases}$$

Figure 22: Firm's Short-run Supply Curve



Finally, recall that the market supply curve is just the sum of the firm's supply curves.

3.3 Long-run Analysis

In the long-run, all factors of production are variable; nothing is fixed. We need to consider two cases, the first where firms are earning positive profits in the short-run and the second where firms are earning negative profits (again, losses) in the short-run. An individual firm (perfectly competitive or otherwise) produces in the long-run only if it covers its opportunity costs, i.e., $\pi \geq 0$ at q^*. This is equivalent to $TR \geq TC$ and $P^* \geq ATC$ at q^*.

Case 1: Short-run Profits
Remember we're talking about economic profits. If firms are earning positive economics profits, there's a great incentive for firms to enter the market. Remember, by assumption, it is costless to enter or exit a perfectly competitive market. New firms' entering will increase the market supply of the good, thus reducing the market-clearing price of the good. This in turn will reduce firms' profits. The quantity traded in the market will increase.

Case 2: Short-run Losses
If firms are earning negative economic profits, the firms that have very high average variable costs (where the average variable cost is greater than the price they receive) will shut down and exit over time. This will reduce the market supply of the good, thus increasing the price of the good. Those firms remaining in the market will see their losses going down until the profits are zero.

In the long-run in a perfectly competitive market, entry and exit drive profits of firms to zero. All firms are just covering their opportunity costs.

Internal Returns to Scale and The Long-run Average Total Cost Curve (LRATC)
See section 2.1.1 of these notes for the definitions of increasing, decreasing, and constant returns to scale.

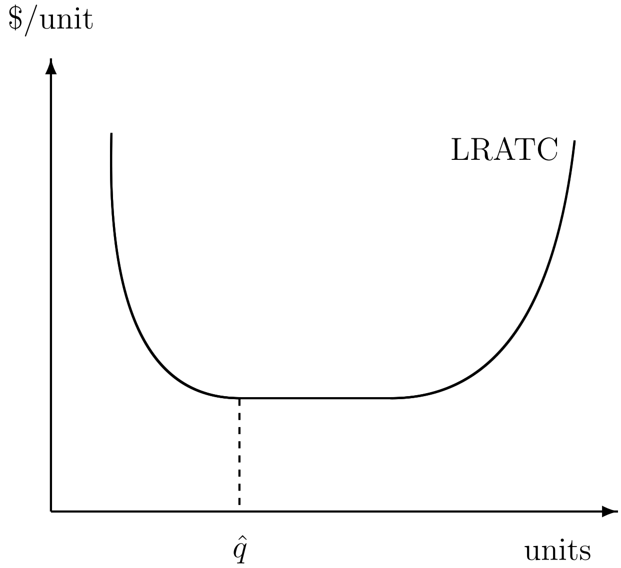
The long-run average total cost curve (LRATC) traces out the short-run total cost curves, showing the lowest average total cost for each quantity produced as the firm expands in the long-run. It is the lower envelope of the short-run average total cost curves.

- If the production function exhibits **increasing returns to scale**, the **LRATC is decreasing** as the firm's output rises.
- If the production function exhibits **constant returns to scale**, the **LRATC is constant** as the firm's output rises.
- If the production function exhibits **decreasing returns to scale**, the **LRATC is increasing** as the firm's output rises.
- Usually, the firm's LRATC is downward-sloping at low levels of output, then constant, then upward sloping as output rises.

The **minimum efficient scale** is the smallest scale of production for which the LRATC is at a minimum, i.e., it's the smallest level of output where the LRATC is constant. In Figure

23, the minimum efficient scale is $\hat{q}$.

Figure 23: Long-run Average Total Cost Curve



External Returns to Scale and the Long-run Market Supply Curve

In the short-run, we found the market supply curve by summing each firm's individual supply curve. In the long-run, though, firms enter and exit markets as the price producers receive changes. So, it is impossible to sum up supply curves since we don't know which firms will be around in the long-run.

The shape of the long-run supply curve depends on the extent to which changes in the quantity of industry output affect input prices. To determine long-run supply, we assume the following:

- All firms have access to the available production technology; output is increased (decreased) by using more (less) inputs, not by technological progress.
- The conditions underlying input markets do not change when the industry expands or contracts.

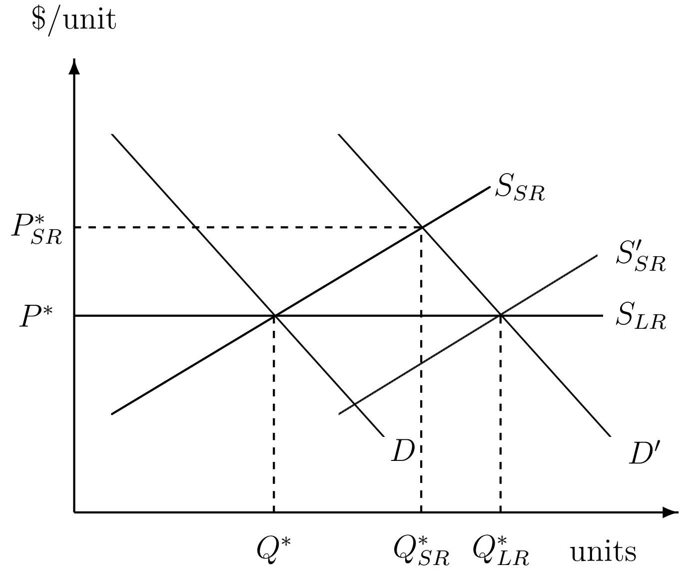
Case 1: Constant Cost Industries

In this case, growth in an industry causes no change in firms' cost structures because of some factor external to the firm. This causes the long-run supply curve to be horizontal at the market price.

Figure 24 shows what happens in a constant-cost industry when demand increases. Starting from a long-run equilibrium (P^*, Q^*) where firms are earning zero profits, when demand

increases from D to D', the market price rises in the short-run from P^* to P^*_{SR}. Firms currently in the market increase output, causing the **quantity supplied** in the market to rise from Q^* to Q^*_{SR}. The short-run equilibrium is (P^*_{SR}, Q^*_{SR}). At this higher price, these firms are earning positive economic profits. This induces other firms to enter the market, causing the market supply curve to shift to the right from S_{SR} to S'_{SR}. This causes the market price to fall and **quantity demanded** to rise to Q^*_{LR}; the quantity supplied by firms originally in the market falls. Because we're assuming input prices are unaffected by the increased output of the industry, entry occurs until the market price falls to the original equilibrium market price. In the new long-run equilibrium (P^*, Q^*_{LR}), the firms in the market originally are producing the same quantity as they were originally.

Figure 24: LR Supply Curve, Constant Cost Industry

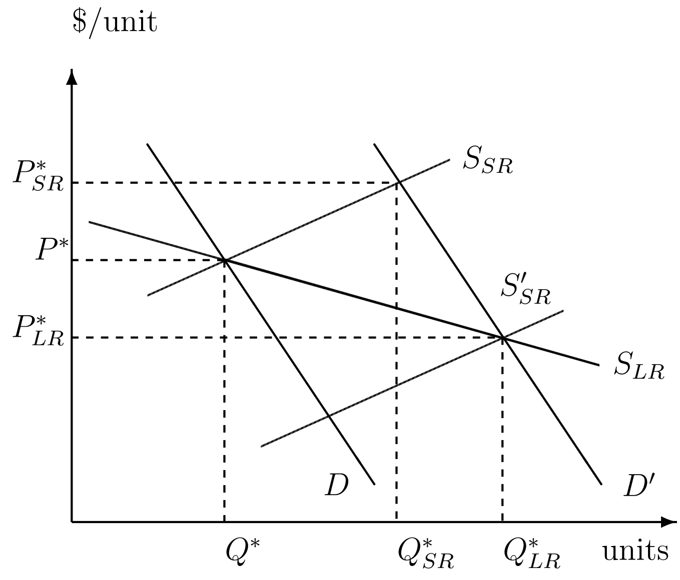


Case 2: External Economies of Scale (Decreasing Cost Industries)

In this case, growth in an industry causes firms' costs to fall because of some factor external to the firm. This causes the long-run supply curve to be downward-sloping.

An increase in demand causes output to rise and the price to rise in the short-run. The new short-run equilibrium is (P^*_{SR}, Q^*_{SR}). Firms in the industry earn positive profits and thus induce new firms to enter the industry. As firms enter, input prices (and thus) costs fall. Supply in in the market increases so much that the market price of the product falls below the original equilibrium price (P^*). The lower market price and lower average cost of production induce a new long-run equilibrium with more firms, more output, and a lower market price.

Figure 26: LR Supply Curve, Decreasing Cost Industry

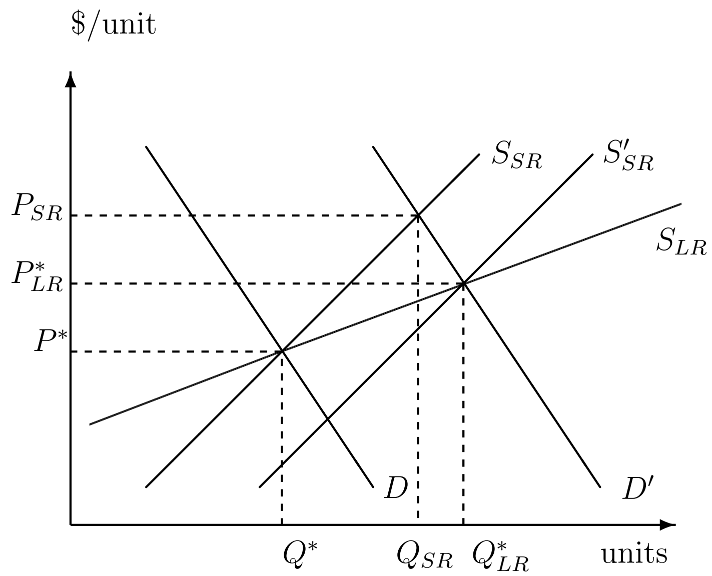


Case 3: External Diseconomies of Scale (Increasing Cost Industries)

In this case, growth in an industry causes firms' costs to rise because of some factor external to the firm. This causes the long-run supply curve to be upward-sloping.

An increase in demand causes output to rise and the price to rise in the short-run. The new short-run equilibrium is (P^*_{SR}, Q^*_{SR}). Firms in the industry earn positive profits and thus induce new firms to enter the industry. As firms enter, input prices and thus costs rise. Supply in the market increases, but the new long-run equilibrium price (P^*_{LR}) exceeds the original long-run equilibrium price. The higher market price and higher average cost of production induce a new long-run equilibrium with more firms, more output, and a higher price.

Figure 26: LR Supply Curve, Increasing Cost Industry



Next, we can use these tools to analyze the effects of imposing a per-unit tax on a market. Normally, we think that demand in the long-run is more price elastic than it is in the short-run. However, this complicates graphical analysis. Since we want to focus on the supply curve, assume that the demand curve is fixed, i.e., the short-run demand curve is the same as the long-run demand curve.

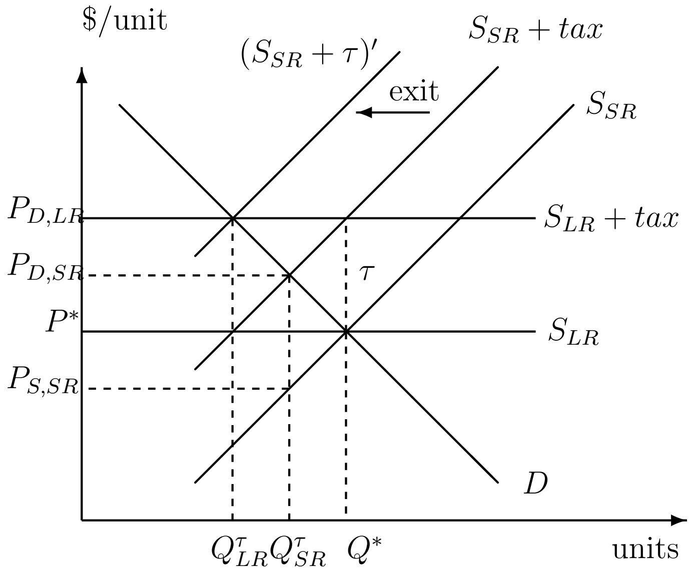
Analysis of Per-Unit Tax in a Constant Cost Industry

For this analysis, I will assume that the industry in question is a constant cost industry. I will start at the long-run equilibrium. When the government imposes a tax, we saw in the short-run, the price buyers pay rises, the price sellers receive falls, and the quantity traded in the market falls. This causes the firms in the market to earn negative profits. Over time, some firms will exit. This causes supply to decrease over time, which drives up the price buyers pay (and the price sellers receive), and the quantity traded in the long-run will be less than it is in the short-run.

Graphically, this is shown in Figure 27. We start at the original long-run equilibrium (P^*, Q^*). The government levies a tax of $\$\tau$/unit. From before, the price buyers pay equals the price sellers receive plus the per-unit tax: $P_D = P_S + \tau$. Assuming the firms remit tax revenues to the government, we shift the short-run supply curve up vertically by τ. In the short-run, the quantity traded will be Q^τ_{SR}, buyers will pay $P_{D,SR}$, and sellers will receive $P_{S,SR}$.

To see what happens in the long-run, we shift the long-run supply curve up vertically by τ. Firms exit over time, and we arrive at the new long-run equilibrium. The quantity traded is Q^τ_{LR}, buyers pay $P_{D,LR}$, and sellers receive $P_{S,LR}$. We can see that since the long-run supply curve is horizontal, i.e., perfectly elastic, buyers bear the full burden of the of the tax: $P_{D,LR} = P^* + \tau$ and $P_{S,LR} = P^*$.

Figure 27: Per-Unit Tax, Constant Cost Industry



I leave it to the reader to analyze the effects of levying a tax in an increasing cost industry or a decreasing cost industry and to analyze the effects of bestowing a per-unit subsidy or imposing a price control on an industry that exhibits constant costs, increasing costs, or decreasing costs.