Control and Estimation with Threshold Sensing for Inertial Measurement Unit Calibration using a Piezoelectric Microstage

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Abstract—A threshold sensing strategy for improving measurement accuracy of a piezoelectric microactuator in calibration of miniature Inertial Measurement Units (IMUs) is presented. An asynchronous threshold sensor is hypothesized as a way to improve state estimates obtained from analog sensor measurements of microactuator motion. To produce accurate periodic signals using the proposed piezoelectric actuator and sensing arrangement, an Iterative Learning Control (ILC) is employed. Three sensing strategies: (i) an analog sensor alone with a Kalman filter; (ii) an analog sensor and threshold sensor with a Kalman filter; and (iii) an analog sensor and threshold sensor with a Kalman smoother are compared in simulation and single-axis experiments. Results show that incorporating threshold sensors in a projected low-noise environment based on capacitive sensing will produce high-accuracy velocity measurements at certain fixed angles, while experimental testing with less reliable piezoelectric sensing shows improved estimation accuracy at all velocities and positions.

I. INTRODUCTION

To improve position or rate estimates in a microactuator for micro-inertial measurement unit (IMU) calibration, Kalman smoothing augmented with threshold sensor measurements and iterative learning control (ILC) is proposed. A micro-IMU consists of one or more microelectromechanical system (MEMS) gyroscopes and/or accelerometers. In a certain range of operation, most gyroscopes have a linear or nearly linear relationship between their output signal and angular velocity, with some bias. By measuring the voltage outputs of the gyroscope at two known angular velocities a calibration curve can be determined. Over time, variation in gyroscope operating parameters introduces gain drift (i.e. error in slope) and bias drift (i.e. error in y-intercept) when measuring angular velocity. Hence, an online calibration system is desirable for long-term accurate measurements by a micro-IMU. One class of proposed miniature calibration systems consists of a small actuator or stage that can produce repeated, prescribed angular velocities or accelerations to a gyroscope payload. This would be comparable to a rateable used to apply external calibration inputs in laboratory environments. In the system targeted here, the required excitation is provided by a 6 DOF piezoelectric actuator, shown in Fig. 1(a), capable of three-axis translation and 3-axis rotation. It is an advanced version of that developed by Aktakka et al. in [1]. The IMU is mounted on a platform which is actuated by the 6 DOF actuator. Such an actuator coupled with precise motion estimation, shown schematically in Fig. 1(b), would not only provide calibration curves, but would also allow identification of cross-axis sensitivity, though this work examines only a single axis.

Fig. 1. (a) Piezoelectric micro-stage prototype to be used in gyroscope calibration; (b) block diagram describing calibration architecture comparing estimated micro-stage position to gyroscope output.

Multi-position calibration [2] and continuous calibration are two methods for calibrating IMUs on a moving platform, which are compared in [3]. Multi-position calibration is mainly used in laboratories and field tests and has a drawback in terms of locking errors (self locking is a part of the calibration process in this method). In the continuous calibration method, used here, the moving platform is used to generate periodic trajectories to excite various error coefficients. Examples of such a method can be seen from earlier researchers [4], with additional previous works done on choosing an optimal trajectory, as in [5].

A continuous calibration process requires very accurate knowledge of platform trajectories as they are experienced by the IMU. This can be achieved relatively easily in an off-line, large-scale setup in a lab [6]. From a control design point-of-view the requirement is tracking of a periodic reference signal. Therefore, Iterative Learning Control (ILC) emerged as a promising control strategy [7]. However, to meet performance requirements for high-accuracy IMU calibration with a micro-scale actuator, feedback from a noisy analog sensor on the piezoelectric stage is likely to be insufficiently accurate. This is due to drift in most analog MEMS sensors, such as additional piezoelectric sensing electrodes or conventional capacitive MEMS sensing electrodes incorporated into the piezoelectric stage. To compensate, a proposed technique is to add high-precision threshold sensors that indicate specific actuator positions with exceptionally high accuracy, though
they are unable to provide other positioning information.

There are different ways to incorporate threshold sensing into estimator design. One is to assume the sensor noise to be state dependent [8], [9] and consider threshold sensor locations to provide a large reduction in noise. Another is to treat threshold sensor measurements as discrete events, as by Lemmon et al. [10], [11]. Astrom, in [12], used a Kalman filter in an ad-hoc estimation scheme when a sensing event occurred. Similarly, here the Kalman filter is revised at threshold sensing events to improve estimates from analog sensors. The authors have previously examined piezoelectric microactuator performance with limited sensor measurements in [13] and a preliminary version of the current scheme in [14], which is evaluated in detailed simulation and through experimental testing in this paper.

II. System Description

The 6 DoF piezoelectric platform being studied is nominally modeled as six uncoupled linear systems. For the purposes of studying estimation trends in this paper, a single DoF is regulated, which is modeled as a second-order, underdamped linear system. This neglects any explicit coupling effects within the reduced-order model, though these can be introduced into the model as external disturbances. The system is described in state space by

\[
\dot{x}(t) = Ax(t) + Bu(t) + Bw(w(t) + d(t)) \\
y_p(t) = Cx(t) + v(t)
\]

(1) (2)

where \( x \) is system state vector, \( A, B, \) and \( C \) are state, input, and output matrices, respectively, and \( y_p(t) \) is a continuous plant output. It is assumed that both random, \( w(t) \) and deterministic, \( d(t) \), disturbances may be present, through matrix \( B_w \), and that the output is subject to noise \( v(t) \).

For controller design and implementation, dynamics are described in state-space form discretized at sampling rate \( T_s \)

\[
x_p(k+1) = A_p x_p(k) + B_p u(k) + B_w (w(k) + d(k)) \\
y_p(k) = C_p x_p(k) + v(k)
\]

(3)

where subscript \( p \) denotes the discretized plant. Without loss of generality, an inner loop controller may be present in the plant, to improve basic system stability or robustness. Random disturbance \( w(k) \) is assumed zero-mean, white, and Gaussian with covariance matrix \( Q \) and \( v(k) \) is assumed zero-mean, white, Gaussian with variance \( R \). For the operating environment of the target piezoelectric calibration stage, \( d(k) \) is taken to be an unknown periodic disturbance within a known frequency range and bounded amplitude.

In addition, due to the use of piezoelectric actuation in the target system, a hysteresis model is also included,

\[
u(k) = f(u_{nominal}(k), u_{nominal}(k))
\]

(4)

though hysteresis at the voltage levels used by the experimental system is small. For purposes of simulation, the simple hysteresis model

\[
\dot{u}_{nominal}(k) > 0 \Rightarrow u(k) = u_{nominal}(k) + u_{hysteresis} \\
\dot{u}_{nominal}(k) < 0 \Rightarrow u(k) = u_{nominal}(k) - u_{hysteresis}
\]

is used in the plant (unknown to the controller).

A. Threshold sensing

A major limitation for using a micro-actuator to provide calibration excitations to an IMU is that typical sensing capabilities of MEMS micro-actuators are limited in their own accuracy. For example, the simplest position sensors that could be added to the target system are additional electrodes on the piezoelectric layer for piezoelectric sensing, or analog capacitive position sensing from partially overlapping plates on the actuator and substrate beneath it. Measurements made by such sensors can used to estimate states using Kalman filter. However, the accuracy of analog-only sensor strategies provides little if any improvement on existing inertial sensor reliability, as sensing noise is substantial and detection circuitry is subject to drift with temperature and age comparable or worse than that in existing MEMS IMUs.

Therefore, an asynchronous sensing scheme is introduced in conjunction with the analog output \( y_p \) in (3). This consists of pairs of displacement sensors that detect specific displacements very accurately when passed through, hence referred to as threshold sensors in this paper. These sensors can be implemented, for example, by applying current sensing to capacitive plates that reach a maximum capacitance at a specific angle. While absolute measurements of the capacitance between plates are sensitive to circuit drift, the maximum capacitance, and corresponding transition from positive to negative current flow, always occurs at the same angle and nearly instantaneously, with much less sensitivity to noise or sensing circuit variation.

In the simulation model in this paper for which the effectiveness of threshold sensing is evaluated, resolution of the threshold sensor is assumed to be constrained by the resolution of a clock signal measuring the time between threshold events. The clock period, \( T_c \), is taken to be \( N \) times faster than sampling time for control, or

\[
T_c = T_s/N.
\]

(5)

An output measure, \( y_t \), a linear combination of the states,

\[
y_t = C_t x_p,
\]

(6)

experiences a threshold detection event when crossing a known threshold level, \( y_{th} \). Matrix \( C_t \) describes the combination of states being measured, and this measure may or may not be the same measure of the states as continuous output \( y_p \). If a threshold crossing is detected during the \( N \) clock timer increments between two measurements of continuous output \( y_p \), the threshold crossing time, \( n_t \in (0, N) \), is recorded. For increasing \( y_t \), for example, \( n_t \) is defined by

\[
y_t(n_t) > y_{th} \geq y_t(n_t - 1)
\]

(7)

Threshold events will also typically be detected for decreasing \( y_t \), and with multiple sensors can be detected at multiple threshold angles. The only information required by the estimation algorithms is that when the threshold value is crossed, the value for \( n_t \) recorded is assumed taken to
correspond exactly to the time at which \( y(t) = y_{0.0} \). In other words, threshold sensing is performed in open-loop with accuracy equal to the resolution of the clock performing timing, while analog feedback is sampled at a lower rate to allow for A/D conversion, filtering, and computation and is used by the controller to aid in estimation of intermediate values and for real-time or iterative feedback.

B. Controller design
The objective of control and estimation design, given the two types of sensors, is to minimize the estimation error between the actual motion experienced by the actuator and state estimates of that motion. Excitation to the IMU is repeated many times to permit averaging of IMU gain estimates for increased accuracy, and thus reference trajectories for the states, \( r(t) \) are chosen to be periodic with at least two different rates (i.e. two different angular velocities for calibration). In addition, the reference trajectory period is taken to be an even multiple of the controller sampling time,

\[
\begin{align*}
    r(t) &= r((t + T) = r((t + N_s T) & (8) \\
    r_p(k) &= r((k + N_s) & (9)
\end{align*}
\]

where \( T \) is the period of the periodic reference and \( N_s \) is the number of slow samples per period.

Due to the periodic nature of the reference trajectories, Iterative Learning Controller (ILC) strategy is used for control purposes. The following are the steps involved in the control signal generation: (a) Initialize reference following with a tracking control designed for the nominal system; (b) At the end of each period of duration \( T \), estimate the states (strategies used for obtaining these estimates are discussed in the following section); (c) Calculate the difference between the reference trajectory and system output at each analog sensor sampling time; (d) Using ILC methodology, update the input signals for the next period using ILC gain \( G_{ILCT} \).

In the ILC method applied, inputs are modified after each iteration (each period in this case) for improved performance in the next period according to

\[
\begin{align*}
    u(N_s + k) &= G_{ILCT} \times [\hat{x}(k + 1) - r((k + 1)) & (10)
\end{align*}
\]

where \( G_{ILCT} \) is a proportional ILC learning function and \( \hat{x} \) is the \textit{a priori} state estimate calculated by a Kalman filter. The goal of the ILC is to mitigate periodic errors during trajectory following, in particular those due to the periodic, deterministic disturbance, \( d(k) \) and hysteresis in the piezoelectric actuators.

III. Estimator Design
Three strategies are compared for estimating the states of the system (step (b) in controller implementation above), which are discussed in the following three subsections. In section III-A analog sensing alone is considered. In section III-B threshold sensor measurements are added to improve the Kalman filter estimates and in section III-C the strategy is modified to include Kalman smoother estimates for the period preceding a threshold crossing. The strategies assume that after a sufficient time, the ILC algorithm reduces the effects of the deterministic disturbance and hysteresis, to the point that they may be neglected during estimator design.

A. Strategy 1 : Kalman filter without threshold sensor
In the first strategy, an analog position sensor alone is used. States are estimated based on noisy measurements from the sensor using a standard Kalman filter.

B. Strategy 2 : Kalman filter with threshold sensor
In the second strategy, both an analog sensor and threshold sensor are considered to be available. States are estimated using Kalman filter equations based on measurements of \( y_p \) at all sampling instants. At the sampling instants just after the threshold sensor crossings, the state estimates are modified using the threshold sensor measurements by inserting an additional time step into the Kalman filter between the typical sampling instants.

Standard Kalman filter equations are used to estimate states when threshold events are not present,

\[
\begin{align*}
    \hat{x}_p(k + 1) &= A_p\hat{x}_c(k) + B_pu(k) & (11) \\
    P(k + 1) &= A_pZ(k)A_p^p + B_pQB_wB_p^p & (12) \\
    \hat{x}_c(k + 1) &= x_p(k + 1) - P(k + 1)C_p(C_pP(k + 1)C_p + R)^{-1}(C_px_p(k + 1) - C_p\hat{x}_p(k + 1) + v(k + 1)) & (13) \\
    Z(k + 1) &= P(k + 1) - P(k + 1)C_p(C_pP(k + 1)C_p + R)^{-1}C_pP(k + 1) & (14)
\end{align*}
\]

with \( \hat{x} \) indicating \textit{a priori} state estimates, \( \hat{x}_c \) indicating \textit{a posteriori}, corrected state estimates, and \( P \) and \( Z \) being \textit{a priori} and \textit{a posteriori} error covariance matrices.

Now consider a sample period in which a threshold crossing is detected. Let \( k^- \) denote the slow-sampled time step before the crossing is detected, \( k^+ \) denote the time step after the crossing is detected, and \( k_i \) denote the new time step occurring at the threshold detection, or

\[
    k^- < k_i = k^- + \frac{N_t}{N_s} < k^+.
\]

In this notation, though \( k_i \) is not an integer, it serves to mark a discrete time point between the ordinary sampling instants.

Then, let \( A^- \), \( B^- \) and \( B^+ \) be the dynamic state matrices of the system discretized with a sampling time equal to the time between \( k^- \) and \( k^+ \) the threshold detection, \( T_s \times N_t \), and \( A^+, B^+ \) and \( B^+ \) be the dynamics matrices of the system discretized with a sampling time of the remainder of the standard sampling period, \( T_s - T_s \times N_t \). These dynamics represent the system behavior before and after threshold crossing as though the system had time varying dynamics from the perspective of the Kalman filter. Using these matrices, state estimation \( \hat{x}(k_i) \) and covariance \( P(k_i) \) at the threshold crossing time can be calculated as

\[
\begin{align*}
    \hat{x}(k_i) &= A^-\hat{x}_c(k^-) + B^-u(k^-) & (16) \\
    P(k_i) &= A^-Z(k^-)A^- + B_wQB_w & (17)
\end{align*}
\]
Strictly speaking, disturbance covariance $Q$ should be also be adjusted for time period, but the effect on estimation was found to be small enough in simulation and experimental testing that this step is omitted for convenience.

Using the threshold level $y_{t0}$ equations in (11) are modified to obtain posteriori estimates at $k_t$, $\tilde{x}_c(k_t)$ and $Z(k_t)$,

\[
\dot{\tilde{x}}_c(k_t) = \tilde{x}(k_t) + P(k_t) \\
\times C_p |C_p P(k_t) C_p^T|^{-1}(y_{t0} - C_p \tilde{x}(k_t)) \tag{18}
\]

\[
Z(k_t) = P(k_t) C_p |C_p P(k_t) C_p^T|^{-1} C_p P(k_t) \tag{19}
\]

After obtaining posteriori estimates at the threshold crossing time, these estimates can be used to obtain a priori estimates for the $k^+$ time step associated with the current crossing,

\[
\dot{\tilde{x}}_p(k^+) = A^+ \dot{x}_c(k_t) + B^+ u(k^-) \tag{20}
\]

\[
P(k^+) = A^+ Z(k_t) A^{\prime T} + B_p^+ Q B_p^{\prime T} \tag{21}
\]

C. Strategy 3 : Kalman smoother with threshold sensor

Since ILC is used for control input generation, the state estimates based on measurements gathered during one period are used to modify the input sequence of the next period only. However, for calibration the entire time series of data may be used, so using stored measurements to improve the state estimates over the entire period using a Kalman smoother may be advantageous. At the end of each reference period, the last threshold crossing time is determined. Using the state estimates from the threshold crossing time, estimates of the previous states were improved using Kalman smoother equations (22) according to

\[
\dot{x}_T(k_t) = \dot{x}(k_t) + Z(k_t) \hat{A}_p |P(k + 1)|^{-1} \times (\hat{x}_T(k + 1) - \dot{x}(k + 1)) \tag{22}
\]

where $k$’s value decreases from the last threshold crossing time to beginning of the period and $\dot{x}_T$ represents the improved smoother estimates, similar to [15].

IV. Simulation results

In simulation, threshold crossing is checked for at each time instant of the discretized plant dynamics. Whenever a threshold crossing is sensed, a threshold sensor loop is initiated beginning with the actual states of the plant at the time instant just before the threshold crossing. During the threshold sensor loop, dynamics of the plant run at a higher sampling rate to detect the more accurate threshold crossing time. Using a threshold crossing time the regular Kalman filter estimates are updated which were estimated based on capacitive sensor measurements. These equations were run at the same sampling time as the nominal plant.

To compare the relative performance of the estimation scheme with and without threshold sensing and ILC, a prototype piezoelectric stage is simulated. The nominal second-order dynamics have a natural frequency of approximately 1.2 kHz and damping ratio of 0.05. For simulation analysis, the range of deterministic disturbances is 5Hz to 5kHz with maximum amplitude 5% of the controlled input, and measurement noise was 17% of the signal amplitude, based on a capacitive sensing circuit model.

The reference trajectory used for preliminary calibration simulations includes a combination ramp (between 0 radians and pi/180 radians or between 0 degree and 1 degree) and step signals at a desired frequency as shown in Fig. 2. A typical steady state response of the system after ILC has performed its learning is shown in Fig. 2. Using the analog and threshold sensors, measurements are taken which are used for estimating states. As noted previously, analog sensors are assumed to be noisy with gaussian distribution, with potential error in their sensor gain (i.e., error in output matrix $C_p$ of the state-space model). Threshold sensors are assumed to be placed at two locations between 0 and pi/180 radians. After each period of the trajectory signal (0.5 milliseconds in this case) errors were calculated between estimated states and reference trajectory. Using the ILC gain and these estimated errors, the input for the next time period was calculated. In order to find the best learning function $G_{ILCT}$ of (10), a set of values between 10 and 200 were applied and the value producing minimum tracking error was selected ($G_{ILCT} = 95$). With over correction, errors were seen to diverge, while with the optimal learning function in the tested range error decreased to a steady-state, small value. The input was initiated at zero in each simulation.

To analyze performance, two threshold sensors were hypothesized to be available at 0.2 degree and 0.65 degrees (or 0.0035 radians and 0.0113 radians). A reference signal of period 0.5 millisecond was applied for 100 time periods. It was found that the response reaches a steady state after around 80 time periods. Average error between the actual states and estimated states as well as average angular velocity and average angular positions were calculated for last 20 time periods. When evaluating estimation performance, the average angular velocity at the $i$’th time step in each reference period after ILC convergence was estimated, and compared to the average state errors at the same time steps.
Fig. 3. Comparison of average angular velocity error for three sensing strategies while threshold sensors are placed at 0.0035 radians and 0.0113 radians.

Fig. 4. Comparison of average angular velocity error for three sensing strategies while threshold sensor is placed at 0.0113 radians alone.

Two cases are compared in Fig. 3 and Fig. 4. In the first case two threshold sensors are used and in the second case only one is used. It was observed that in the first case, (Fig. 3) at two locations the average angular velocity error is very small (on the order of $10^{-5}$) for both threshold sensing controllers compared to analog sensing alone (Fig. 3). When one of the threshold sensors is removed, the performance deteriorated and error value increased to the order $10^{-4}$ as shown in Fig. 4, becoming comparable to the capacitive sensor alone case, as would be expected from the inability to effectively scale output measurements using only a single threshold sensor. By simulating different threshold sensor locations, it was observed that depending on the threshold locations the lowest error locations change. However, there were consistently two angular positions which offered very small average angular velocity error, and these positions remained fixed over various simulation conditions for a specific pair of threshold angles.

While the ILC controller is not explicitly accounted for in Kalman filter design, its presence tends to improve estimation accuracy at the highest accuracy estimation locations when there is model uncertainty, as shown in Table I. When model knowledge is perfect, ILC provides no additional benefit (Case 2 vs. Case 1) and in fact can cause greater error in estimates when using threshold sensing. However, once model uncertainty is present (Cases 3 and 4), use of ILC does produce improved estimation accuracy, and the threshold sensors likewise improve angular velocity estimates as anticipated. For a sample target gyroscope [16], the angular rates with minimum velocity error and their corresponding error levels can be used to predict calibration accuracy, with best estimation accuracies on the order of 10 to 100 ppm.

V. EXPERIMENTAL RESULTS

To validate the proposed algorithm experimentally, a prototype piezoelectric stage was tested in rotational oscillation about its $x$-axis. The prototype stage was instrumented only with piezoelectric displacement sensing, rather than capacitive sensing, due to its production by a less-complex fabrication process than stages for full gyroscope calibration. As such, the stage lacked on-board threshold sensing. In its place, a helium-neon laser was reflected off the stage surface to a high precision On-Trak light sensing array (LSA) translational sensor. While the LSA is also subject to some drift over time, for estimator validation the LSA response was treated with as a true measurement of stage position, with two angles from the LSA treated as threshold measurements.

The reference LSA measurement trajectory used in the experimental verification was a sinusoidal signal of frequency 685Hz and amplitude of 3V. A system model was identified using the circle fit method of modal analysis (natural frequency=678 Hz, damping ratio=1.8% and gain $\approx$ 79) from piezoelectric measurements and scaled to match the rotational amplitude measured by the LSA. The covariance values used for kalman filter design are initial state covariance...
$P(0) = [1 \ 0; 0 \ 1]$, disturbance covariance $Q = 1$ and noise variance $R = 10$. Fig. 5 compares the LSA output, threshold sensor algorithm estimates and estimates obtained without threshold sensing, indicating threshold sensing points with black circles. In the filtering algorithm the threshold value improves the velocity after the threshold sensing location and in the smoother algorithm it improves accuracy on both sides. A low pass filter is used to filter out some of the high frequency noise in these estimates. The proposed algorithm estimates the derivative of the LSA output well near the threshold sensing points. Both piezoelectric and LSA angular measurements are less accurate than capacitive sensing, which is to be ultimately incorporated into the piezoelectric stage, but substantial improvements in angular velocity error are seen. These errors range from 30% to 18% using piezoelectric sensing alone, versus approximately 5% to as low as 0.2% at maximum accuracy positions when certain threshold angles taken from the LSA are used to improve the estimator.

VI. DISCUSSION AND FUTURE WORK

A strategy for controlling a piezoelectric actuator to generate periodic reference trajectories for calibrating an IMU has been presented. The chief contribution of this paper is to propose and evaluate through simulation and experimental studies the impact of incorporating a novel threshold sensing strategy in an iterative learning control algorithm. Such threshold sensors would provide very accurate angular velocity estimates at certain locations, with much less sensitivity to actuator variation over time than traditional analog sensing. It was shown that by using two of such accurate points, certain angular positions and/or rates of the simulated piezoelectric stage can be known very precisely, even in the presence of unmodeled hysteresis, external vibration disturbances, and analog sensor modeling error.

The implications of the results obtained by simulation are that adding threshold sensing to the ILC algorithm can produce exceptionally accurate measurements of certain displacements and/or angular rates of a micro-actuator, though improvements in position estimation at other actuator positions are very limited. Experimental testing of the algorithm also shows substantial estimation improvement with threshold sensing events, though the instrumentation of current piezoelectric actuators is not accurate enough to push estimation accuracy to that anticipated with capacitive position sensing. In future work, angular velocity estimation will be tested on piezoelectric stages with capacitive sensing, currently being fabricated, along with control of additional axes of motion.

REFERENCES